

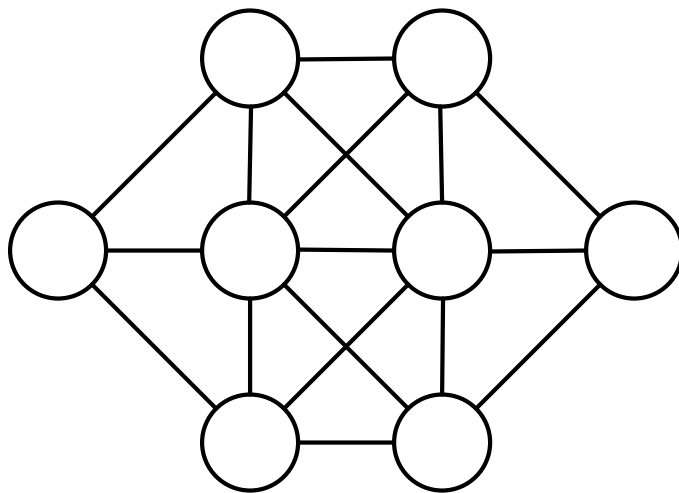
Coláiste An Spioraid Naoimh Maths Circle
Lesson 3

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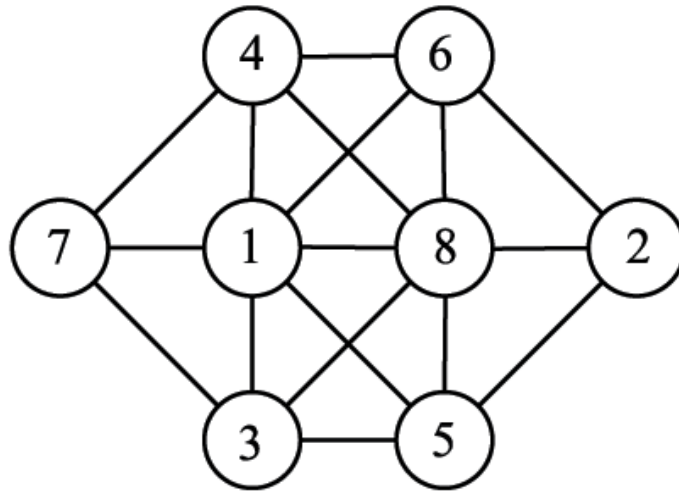
October 26, 2012

Last week's take home problem

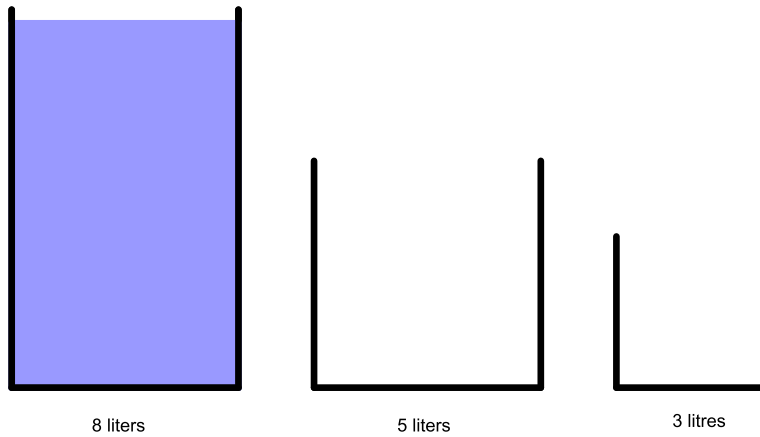
Place the numbers 1 to 8 in each of the circles such that no 2 consecutive numbers are in circles joined by lines.



Solution: The trick to this problem is to notice that 1 and 8 both only have one number that they are not allowed to be joined to, but every other number has two. The two circles in the centre are joined to every other circle except one. So, 1 and 8 have to go in the central circles and the solution follows.



1. **Water container puzzle** You are given 3 water containers- One holds 8 liters and is full of water. The others hold 5 liters and 3 liters and they are empty. All three containers are unmarked.



The object of the puzzle is to divide the 8 liters into two parts of 4 liters each. The difficulty in this problem, is that we can not measure what one liter is. We have a measure of 3 liters and 5 liters (knowing 8 liters is useless, as this is all the water)

solution:

8	0	0
5	0	3
5	3	0
2	3	3
2	5	1
7	0	1
7	1	0
4	1	3
4	4	0

or:

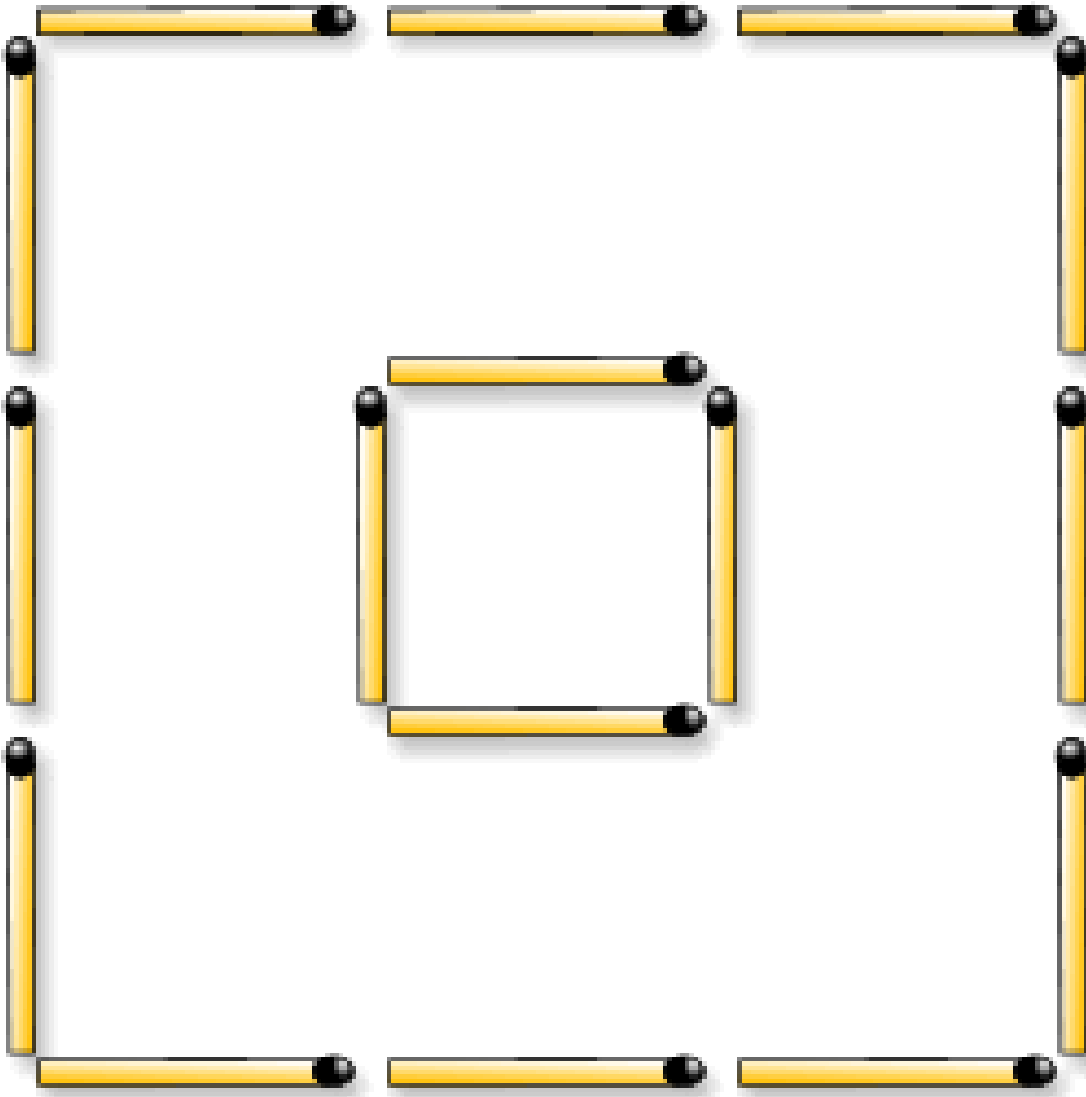
8	0	0
3	5	0
3	2	3
6	2	0
6	0	2
1	5	2
1	4	3
4	4	0

Now suppose you have 3 containers, that hold 8 liters, 4 liters and 2 liters, again, with the 8 liter container being full and the other two empty. Try to divide the water into 2 parts- 5 liters and 3 liters.

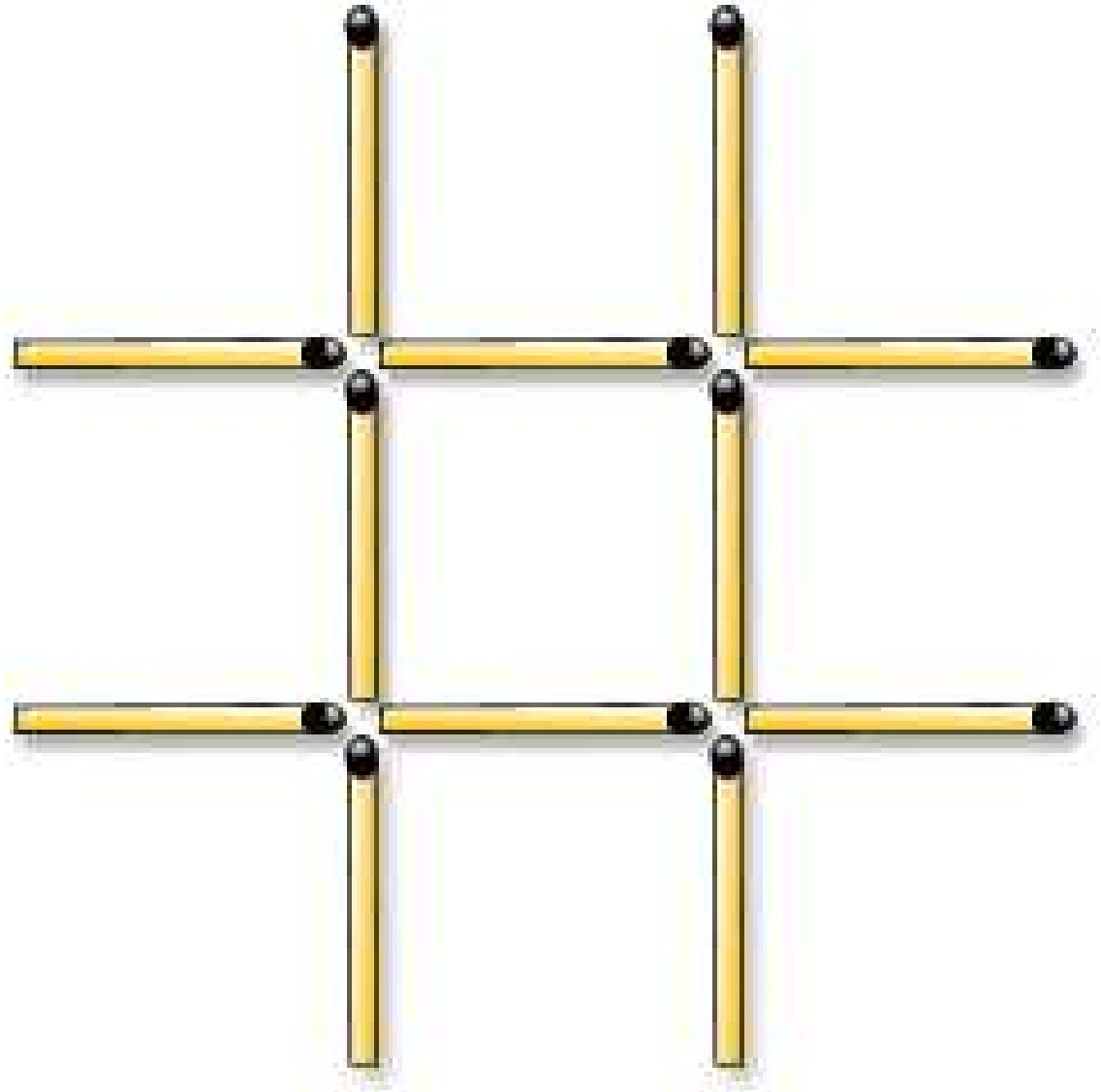
This is impossible! Because each of the containers hold an even number of liters, when we transfer water between containers we are just adding and subtracting even numbers, and so the result is always even, so we can not get an odd number of liters, such as 3 or 5.

2. Match stick puzzle

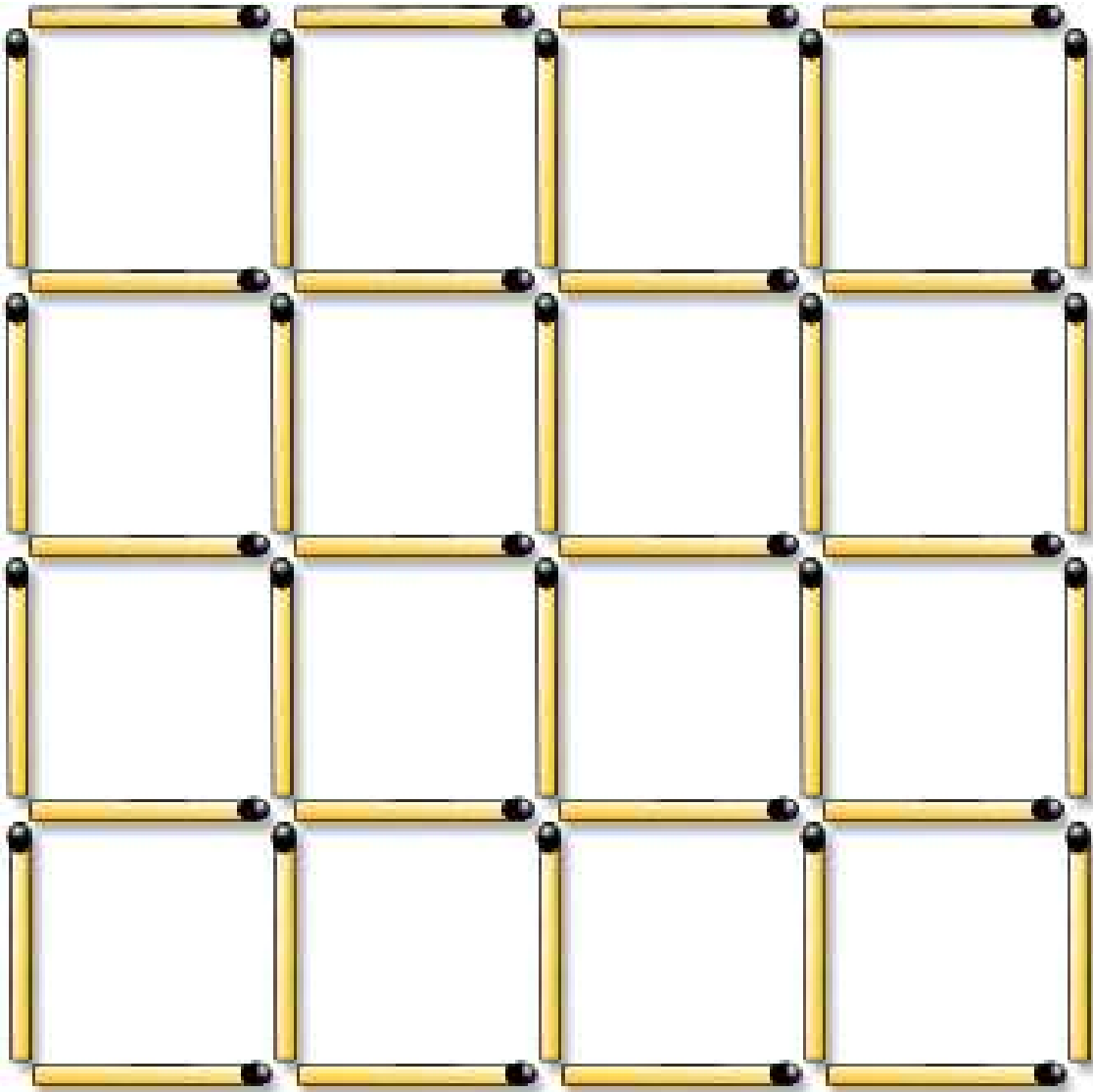
(i) Move four matches to form three squares:



(ii) Move three matches to form three squares:

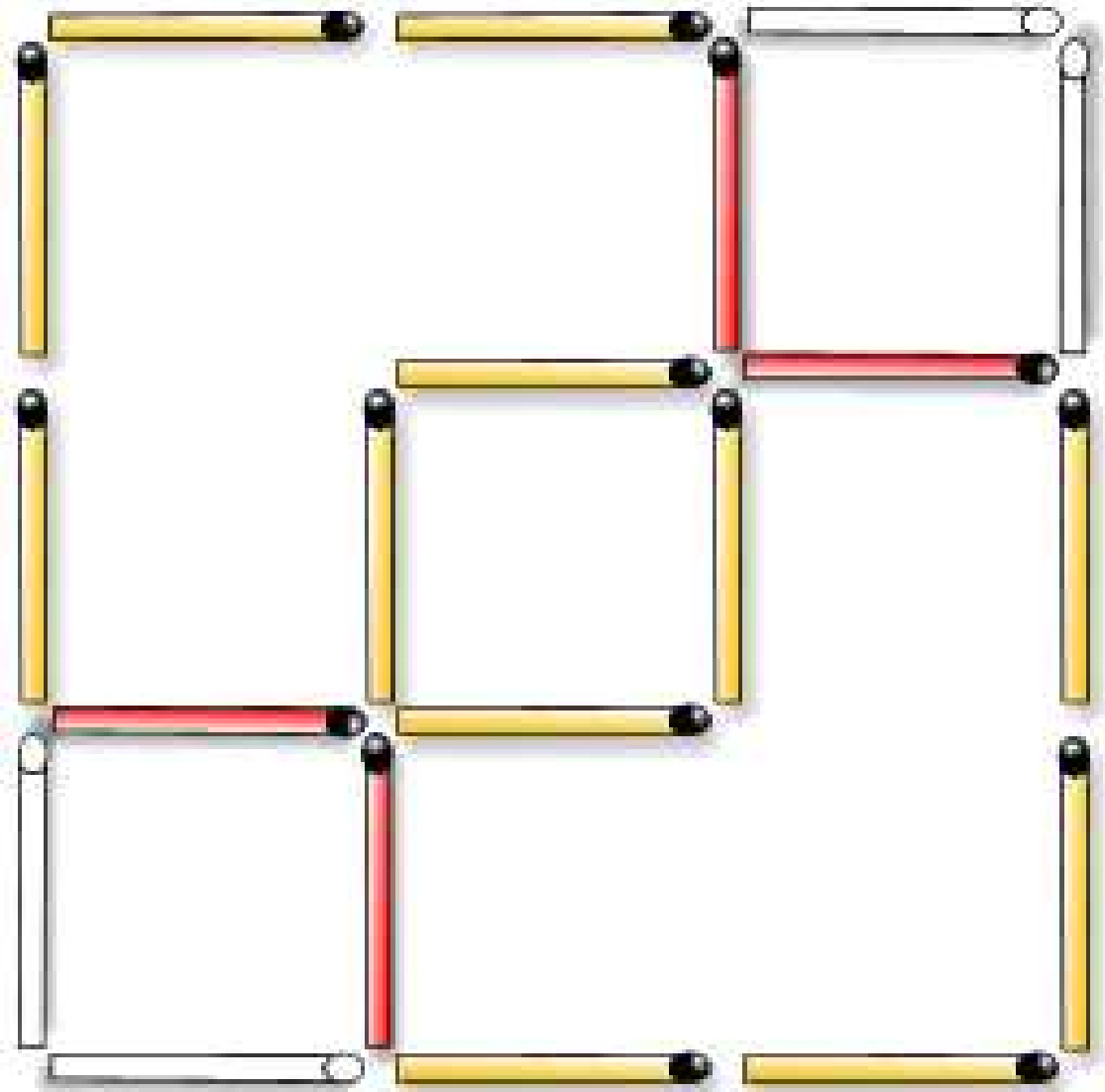


(iii) Remove nine matches so that there are no squares remaining:

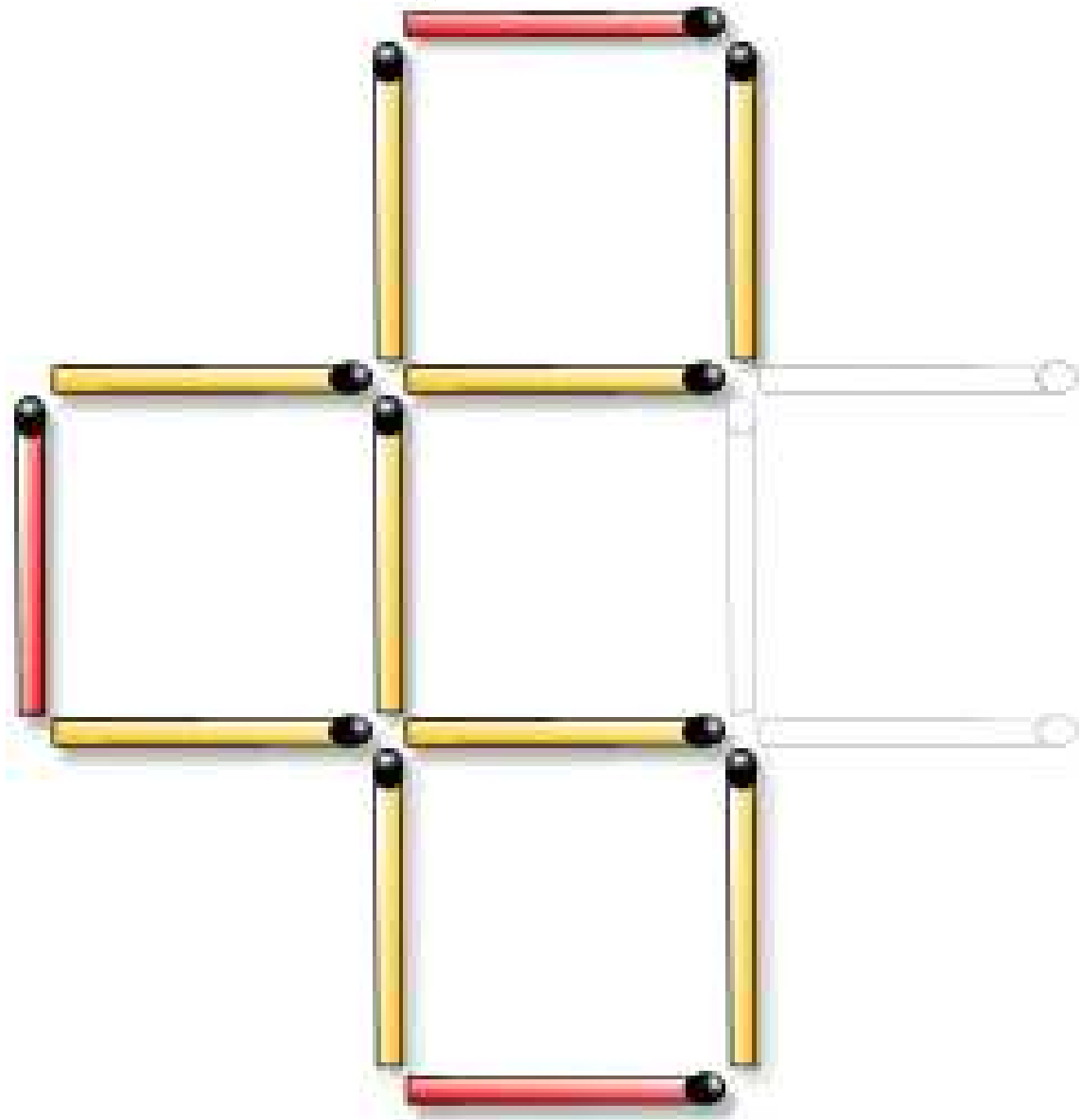


Solutions

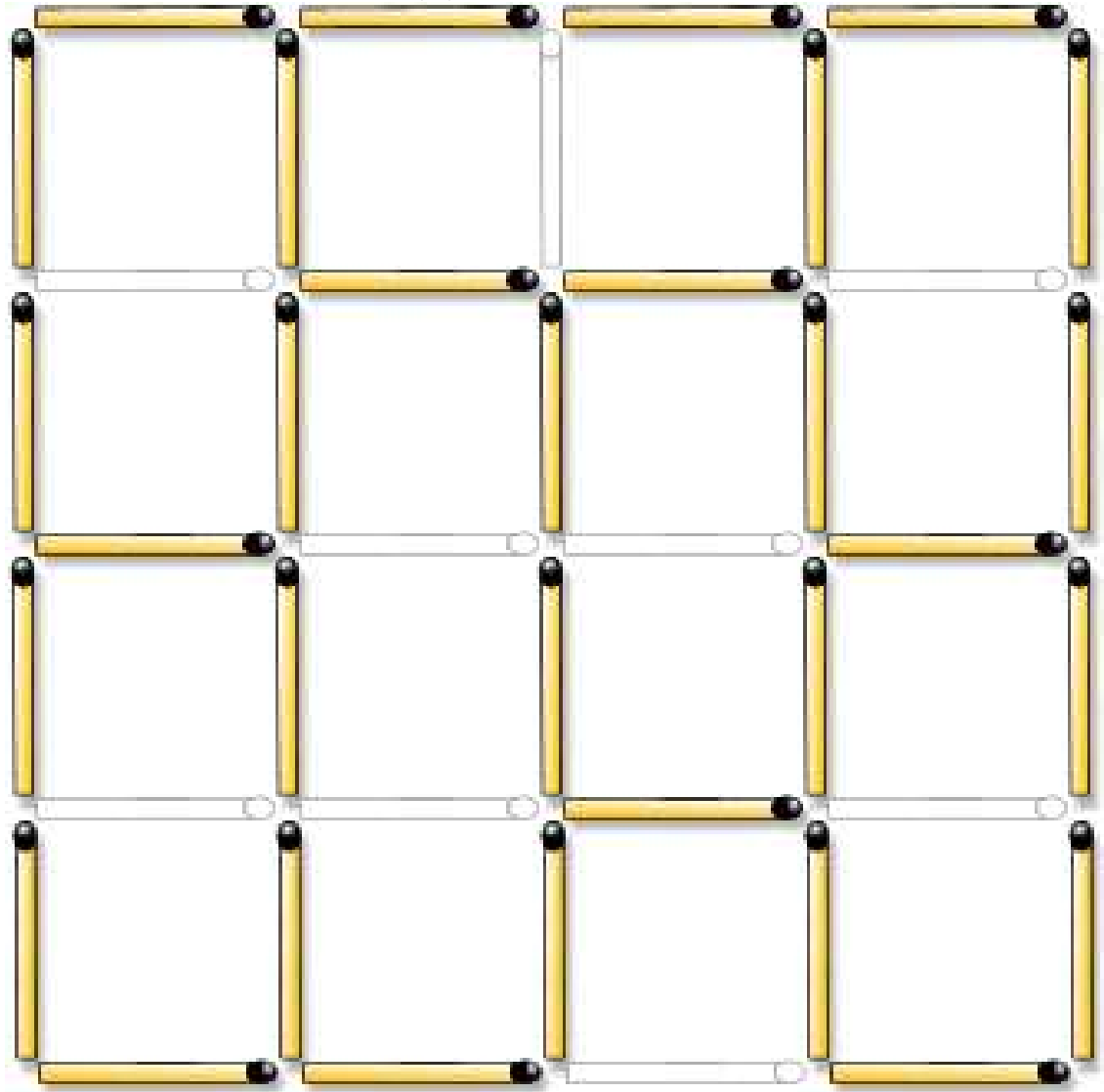
(i)



(ii)



(iii)



3. Nim

Nim is a game for two players. 32 match sticks are placed in a heap in the middle of the table. Players take turns of removing 1, 2 or 3 match sticks. The player who removes the final stick wins.

- Is it better to go first or second? What is the best strategy?
- What if there is a different number of match sticks in the starting pile- for example 35. Who has the advantage now? What should their strategy be?
- What if the number of match sticks you are allowed removed was different- for example up to 5. What should the strategy be now?

Solution The person going second can always win. On every turn if the first person removes x sticks, the second person should remove $4 - x$. This way after his every move there will be a multiple of 4 match sticks taken removed. Because 32 is a multiple of 4 he will be removing the last matchstick.

If the number of match sticks is not a multiple of 4, the player going first has the advantage- He should remove the number of matches to leave a multiple of 4 and then continue as though he was the second player.

Whatever amount of matchsticks you are allowed to remove, the principle still holds- the trick is to always look at multiples of the integer 1 greater than the amount you are allowed to remove.

Take home problems

1. Two Guard Riddle

You face two guards: A knight and a knave. The knight will always tell the truth and the knave will always lie, and you do not know which is which. You must find out, which one is the knight and which one is the knave, and are only allowed to ask one question to one of the guards.

What question should you ask?

2. Eight Snooker Ball Puzzle



Suppose you had eight snooker balls, all identical except one which is slightly heavier. You have a pair of scales, which can give 3 possible readings: left side is heavier, right side is heavier and both sides weigh the same. How do you identify the heavier ball after just 2 weighings?