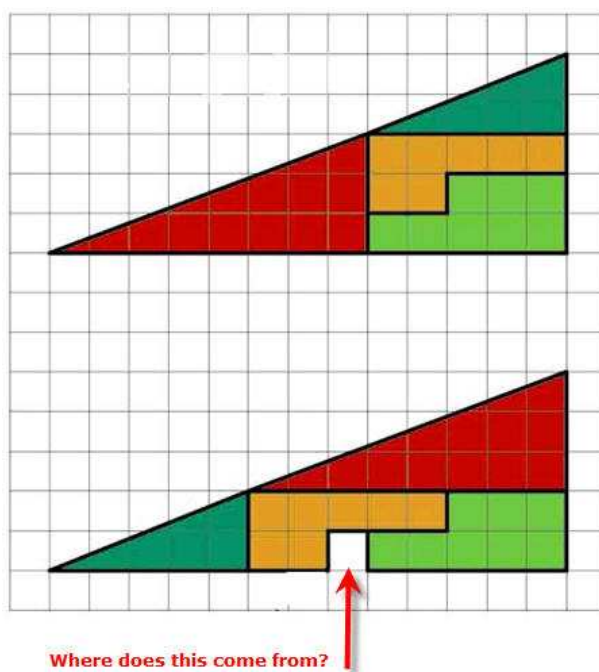


Coláiste An Spioraid Naoimh Maths Circle  
Lesson 8

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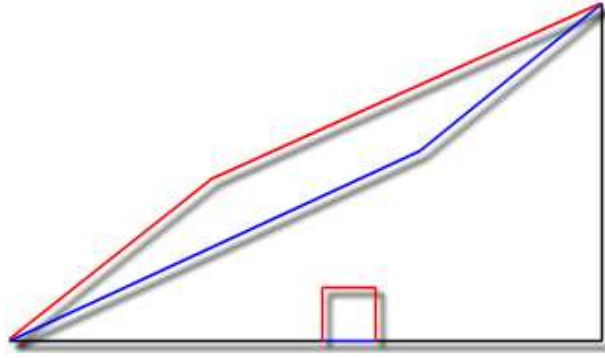
December 7, 2012

**Last Weeks Take-Home Problem**



**Solution:**

The trick with this optical illusion is that the big “triangle” isn’t actually a triangle. The slope of the red triangle is  $\frac{3}{8}$ , whereas the slope of the green triangle is  $\frac{2}{5}$ . The following figure exaggerates the effect:



In summary, the missing square is explained by the fact that in the first “triangle” the hypotenuse bends in and in the second “triangle”, the hypotenuse bends out.

# 1. Mobius Strips



Take a long strip of paper- make a half twist in it and then stick the two ends together. What you now have is called a Mobius strip.

- (a) Draw a line along the strip, halfway between the edge(s), until you get back to where you started. How many sides does the Mobius strip have?
- (b) How many edges does it have?
- (c) How many sides and edges does a strip with 2 half twists have? How about 3? 4? etc.
- (d) Cut along the line that you've drawn. What do you get?
- (e) Make another Mobius strip and this time, cut along the line  $\frac{1}{3}$  of the way in from the edge. What do you get?
- (f) Make a strip with 2 half twists and cut along the line half way between the edges. What do you get?
- (g) Make a Mobius strip with 3 half twists and cut along the line half way between the edge(s). What do you get?
- (h) Can you think of any practical applications of Mobius strips?

**Solutions:**

- (a) The line drawn will pass through the point on the other “side” of the starting point and then back to where it started. This continuous line demonstrates that the Mobius strip has only one side.
- (b) Similarly we can see that there is only one edge.
- (c) All the strips with an even number of half twists have two sides and two edges. All the strips with an odd number of half twists have one side and one edge.
- (d) You will get a long strip with 4 half twists.
- (e) This time, you get two strips, interlocked. One is another Mobius strip,  $\frac{1}{3}$  the width of the original Mobius Strip. The other is a long strip with 4 half twists.
- (f) You should get 2 interlocking strips, each with 2 half twists.
- (g) You will get a strip with 8 half twists, but also tied in a trefoil knot.
- (h) Mobius strips have been used as conveyor belts, so that the wear of each “side” would be equal, and therefore the belt would last longer. They’ve also been used as a continuous loop recording tape to double the playing time. There are also, many other applications.

## 2. Non Transitive Dice



These so called “Non Transitive Dice”, demonstrate a probability paradox that violates common sense and traps the unwary! A student chooses one of the three dice. The demonstrator then chooses another and both dice are rolled and the winner is the person that scores the highest. (best 3 out of 5 might be advisable!).

Students are asked, how are we giving ourself an advantage no matter which of the dice the student chooses. They are asked, which has a better chance of winning, red or green? What about green or blue? What about blue or red?

**Solution:**

	1	4	4	4	4	4
2	G	R	R	R	R	R
2	G	R	R	R	R	R
2	G	R	R	R	R	R
5	G	G	G	G	G	G
5	G	G	G	G	G	G
5	G	G	G	G	G	G

Table 1: Red vs Green

	2	2	2	5	5	5
3	B	B	B	G	G	G
3	B	B	B	G	G	G
3	B	B	B	G	G	G
3	B	B	B	G	G	G
3	B	B	B	G	G	G
6	B	B	B	B	B	B

Table 2: Green vs Blue

	3	3	3	3	3	6
1	B	B	B	B	B	B
4	R	R	R	R	R	B
4	R	R	R	R	R	B
4	R	R	R	R	R	B
4	R	R	R	R	R	B
4	R	R	R	R	R	B

Table 3: Blue vs Red

We can see from making tables that, green outperforms red, blue outperforms green and red outperforms blue. If the students minds are not already blown, they can think about what outperforms what if two of each dice are used instead of one. Amazingly, the order is reversed- red outperforms green, green outperforms blue and blue outperforms red.