Coláiste An Spioraid Naoimh Maths Circle
Lesson 7

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Last Weeks Take Home Problem

Mancala Finishing Moves

The following represents a position in mancala- each of the boxes is one of pits, with the number in them being the number of stones in that pit, and the last box being the mancala.

0	0	0	0	2	1	

This a "finishing position" in Mancala, meaning that you can finish the game from this position. You do this by moving from the first position, then the second position, then the first position. Each time, you get an extra go, because your last stone falls in the mancala:

0	0	0	0	2	1	
0	0	0	0	2	0	1
0	0	0	0	0	1	2
0	0	0	0	0	0	3

Can you find a "finishing position" that has stones in every pit?
Hint: start by finding one with stones in the first 3 positions, then the first 4, etc.

Solution:

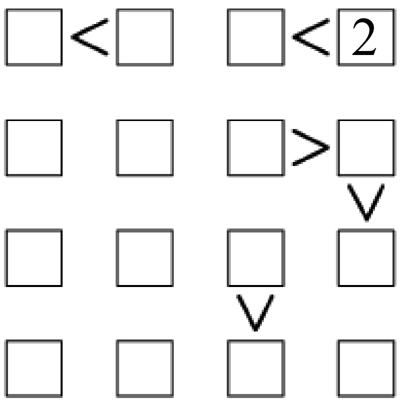
					1
				2	1
			3	1	1
		4	2	2	1
	5	3	1	1	1
6	4	2	3	1	1

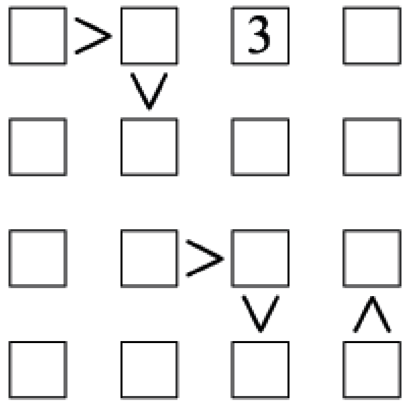
This question is difficult if you start with the case of 6 pits but if you solve it recursively, it's a lot easier. If we know the finishing position with stones in the first n pits, then to find the finishing position with stones in the first $n+1$ pits we need to just find the arrangement that can be moved to the finishing position using n pits.

So, the first thing we need to notice is that for each n, the finishing position using the first n pits must have exactly n stones in the nth pit, so that when we move it, it's last stone drops in the mancala. Every other pit must have 1 less than what it would start with in the finishing position using $n-1$ pits, because when we move the stones from pit n it increases the amount in all the pits in front of it by 1.

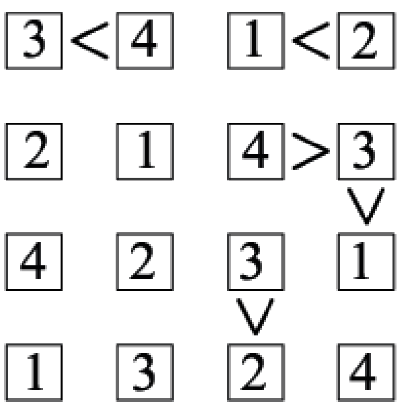
However, the problem say's that there must be stones in **every** pit, so when there are some 1s in the first few pits in the finishing position of $n-1$ pits, we can't replace them by 0's when constructing the finishing position of n pits. Instead we replace them by the finishing position of the same number of pits as the number of 1s.

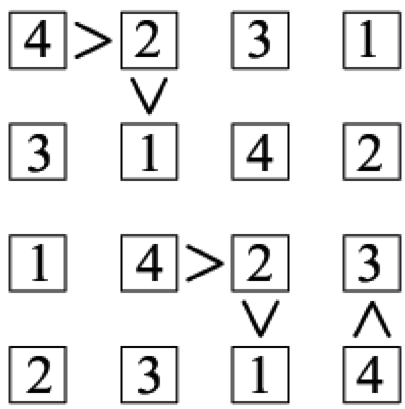
1. **Futoshiki**
   The puzzle is played on a square grid, such as 4 x 4. The objective is to place the numbers 1 to 4 (or whatever the dimensions are) such that each row, and column contains each of the digits 1 to 4. Some digits may be given at the start. In addition, inequality constraints are also initially specifed between some of the squares, such that one must be higher or lower than its neighbour. These constraints must be honoured as the grid is filled out.



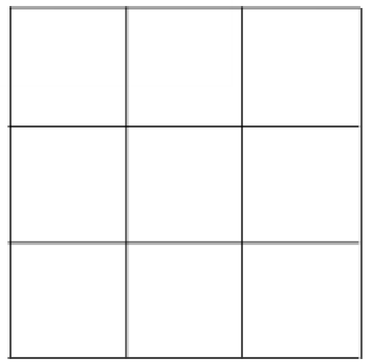


Solutions:

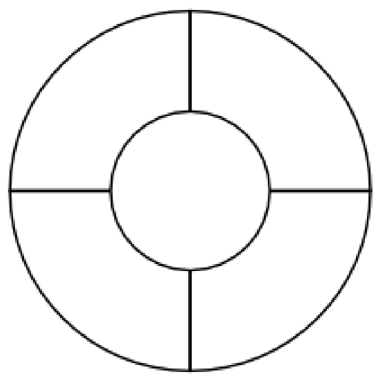




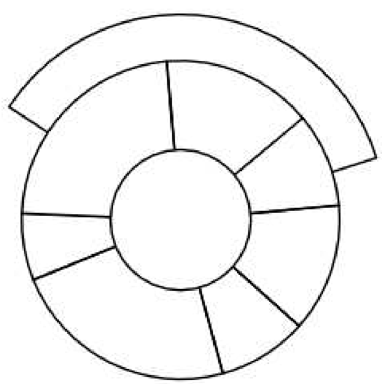
2. **Four Colour Theorem**
For each of the following diagrams, how many colours do you need to use, to colour them in so that no two regions that share an edge are the same colour?



(a)



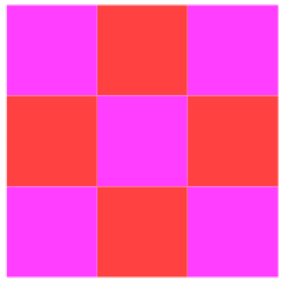
(b)



(c)

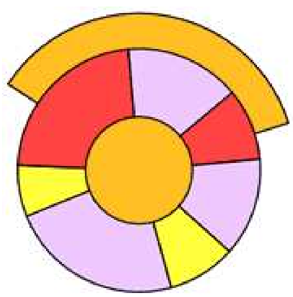
Solutions:

(a) 2 colours



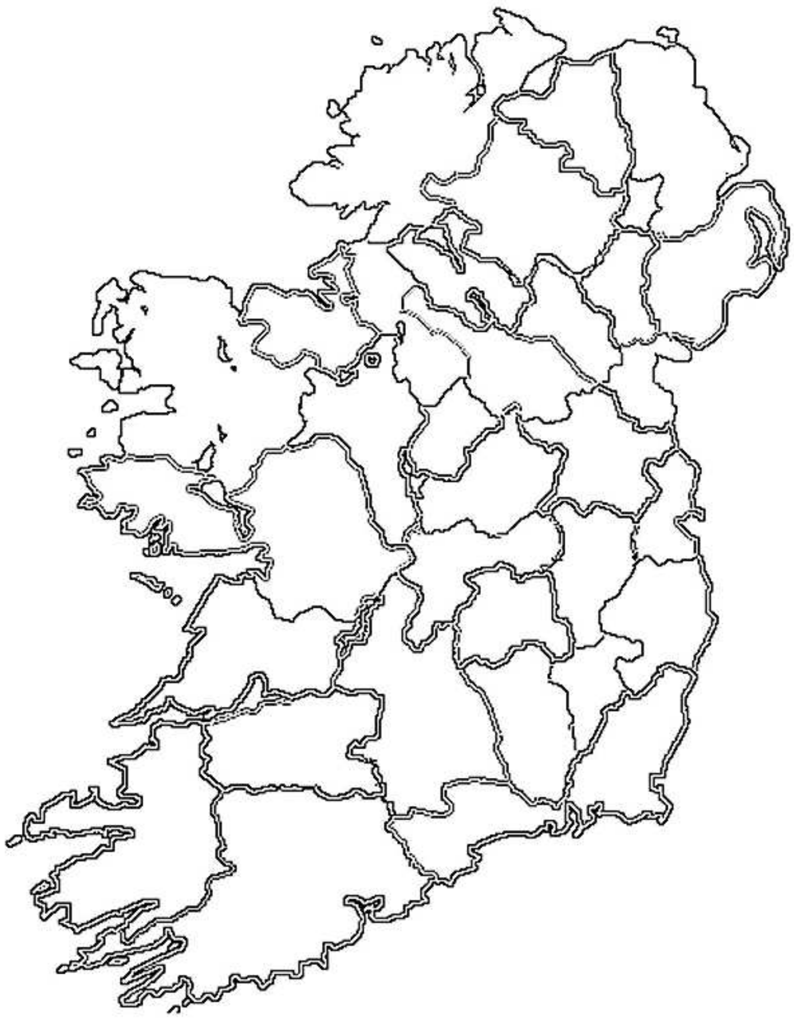
(b) 3 colours

(c) 4 colours



The four colour theorem states that given any map in the plane, no more than 4 colours are needed to colour the regions of the map so that no two adjacent regions are the same colour. The theorem was first stated in 1852 but wasn't proved until 1976- by Kenneth Appel and Wolfgang Haken. It is famous for being the first theorem, whose proof used a computer. Many people would argue that Appel and Haken cheated by using a computer.

Given the map of the counties of Ireland, how many colours are needed to colour all the counties? Can you do it with three colours?



Take Home Problem