

Coláiste An Spioraid Naoimh Maths Circle

Lesson 4

Robert Linehan
Lorcán Mac An tSaoir
Gerry Hyde
Denis O Sullivan

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Last Week's Take Home Problems

1. Two Guard Riddle

You face two guards: A knight and a knave. The knight will always tell the truth and the knave will always lie, and you do not know which is which. You must find out, which one is the knight and which one is the knave, and are only allowed to ask one question to one of the guards.

What question should you ask?

Solution: You should ask one of them, "If I asked you were you the knight, what would you say?"

If you are talking to the knight, obviously he will say "yes".

If you are speaking to the knave, had you asked him if he was the knight he would lie and say "yes"- because you are asking him what he would say, he will lie about his answer and so tell you that he is not the knight.

2. Eight Snooker Ball Puzzle

Suppose you had eight snooker balls, all identical except one which is slightly heavier. You have a pair of scales, which can give 3 possible readings: left side is heavier, right side is heavier and both sides weigh the same. How do you identify the heavier ball after just 2 weighings?

Solution:

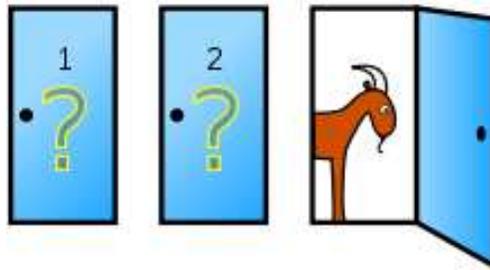
Number the balls 1 through to 8. First we weigh balls 1, 2, 3 against

4, 5, 6.

- If they balance, weigh 7 against 8 to see which is heavier.
- If 1, 2, 3 is heavier than 4, 5, 6, then weigh 1 against 2. If they balance, 3 is the heavy one. If they don't balance, we know which is the heavy one. We use the same method if 4, 5, 6 is heavier than 1, 2, 3.

Warm-Up Problem - Monty Hall Problem

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?



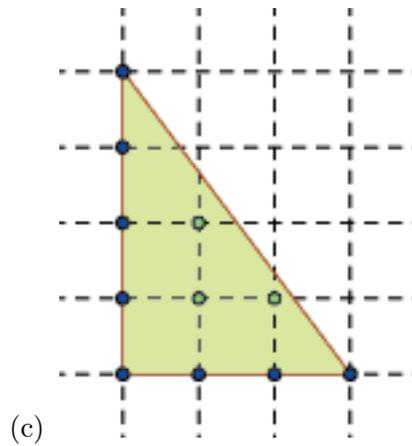
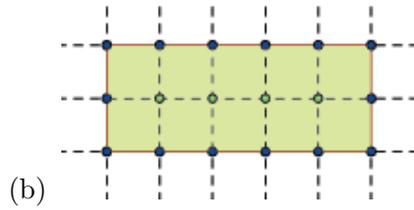
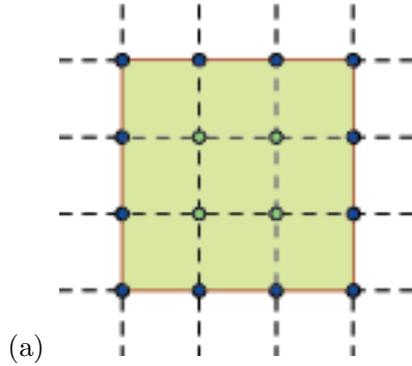
Solution:

It may seem counter-intuitive, but it's actually to your advantage to switch your choice. The easiest way to look at the problem, is that by switching your choice you are betting that your original choice was wrong. Originally you had a 1 in 3 chance of being right, so you had a 2 in 3 chance of being wrong. Therefore, by switching your choice you are giving yourself a 2 in 3 chance of being right.

We will reenact the Monty Hall gameshow using three cups and a ball under one.

1. Pick's Theorem

Calculate the area of each of the following shapes:



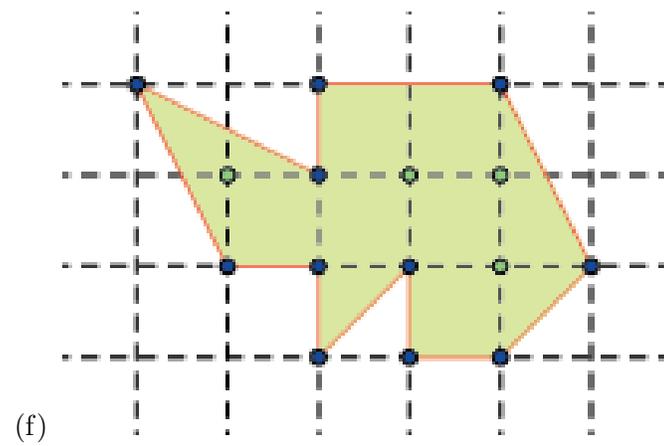
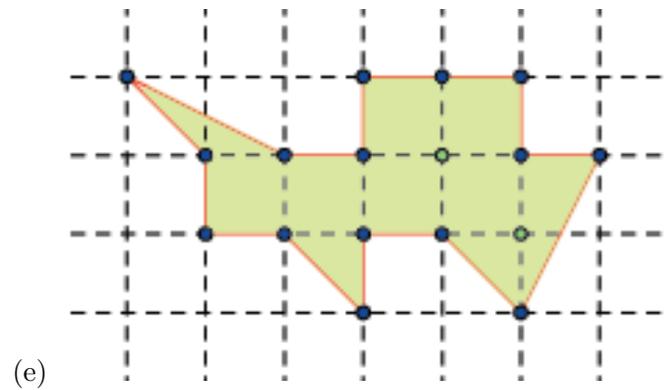
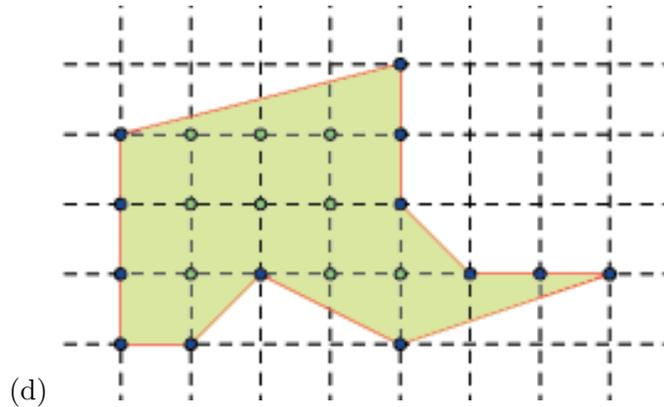
Each of these polygons are "lattice polygons", meaning their vertices all lie on the points of a square lattice. In 1899, Georg Pick found a formula to calculate the area of any such polygon:

$$A = \frac{1}{2}B + I - 1$$

where B = no. of points on the boundary (blue), I = no. of points on the interior of the polygon (green).

Test Pick's formula, by using it to calculate the area's of the polygons above, and checking your answer.

Now use Pick's formula to calculate the areas of the following, more complicated polygons:

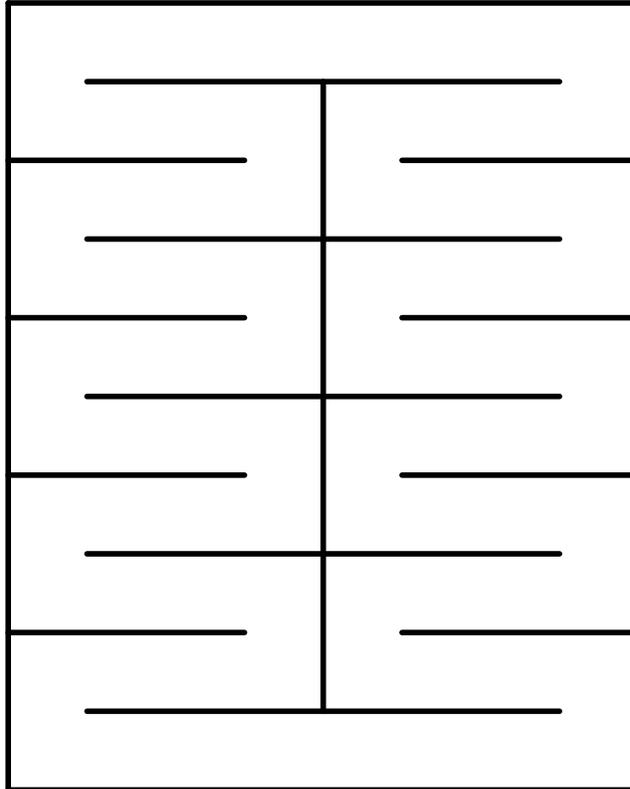


2. Hole in the page

This is a team exercise- the object is to cut a hole in an A4 page, big enough to fit the whole group inside.

Solution:

If we cut along the lines as shown, we will get a big loop to fit everyone inside.



Take Home Problem

A man has to get a fox, a chicken, and a sack of corn across a river. He has a rowboat, and it can only carry him and one other thing. If the fox and the chicken are left together, the fox will eat the chicken. If the chicken and the corn is left together, the chicken will eat the corn.

How does the man do it?