Coláiste An Spioraid Naoimh Maths Circle
Lesson 2

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Last week's take-home problem

When Carl Friederich Gauss was 7 years old he had a lazy maths teacher! One day the teacher didn't feel like teaching at all, and to keep the class busy he told them to add up all the numbers from 1 to 100. In his head, Gauss thought of a very smart way of doing this and was able to shout out the answer in less than a minute! How did he do it?

Solution:

We write out our sum twice, in 2 rows. The second time we write the terms in opposite order, as shown. We pair each term in the first row with the term in the second row and each pair adds up to 101.

$$
\begin{array}{rlllllllllllll}
S = & 1 & + & 2 & + & 3 & + & \dots & + & 98 & + & 99 & + & 100 \\
S = & 100 & + & 99 & + & 98 & + & \dots & + & 3 & + & 2 & + & 1 \\
\hline
2S = & 101 & + & 101 & + & 101 & + & \dots & + & 101 & + & 101 & + & 101
\end{array}
$$

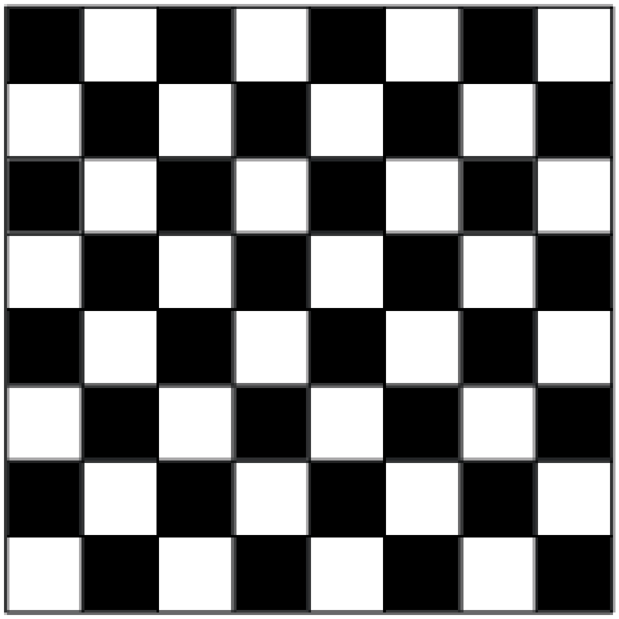
This sum is a lot easier to evalutate:

$$
\begin{aligned}
2S &= 100 \times 101 \\
2S &= 10100 \\
S &= 5050
\end{aligned}
$$

We can generalise this- the sum of all the integers from 1 to n is given by:

$$\frac{(n)(n+1)}{2}$$

1. **How many squares are on a chess board?**



How many squares are on a chess board? **Hint**: the answer is **NOT** 64!!

Solution:

To systematically count all the squares we must count how many of each size square there are. The easiest way of counting them is for each size, count the amount of positions that the bottom left 1×1 square can be in.

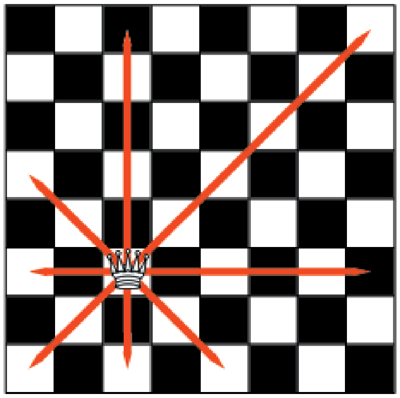
dimension	no of squares
8×8	1
7×7	2^2
6×6	3^2
5×5	4^2
4×4	5^2
3×3	6^2
2×2	7^2
1×1	8^2

We saw that the sum of the integers from 1 to n can be given by $\frac{(n)(n+1)}{2}$. We also have an expression for the sum of the squares of the first n integers:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{(2n+1)(n)(n+1)}{6}$$

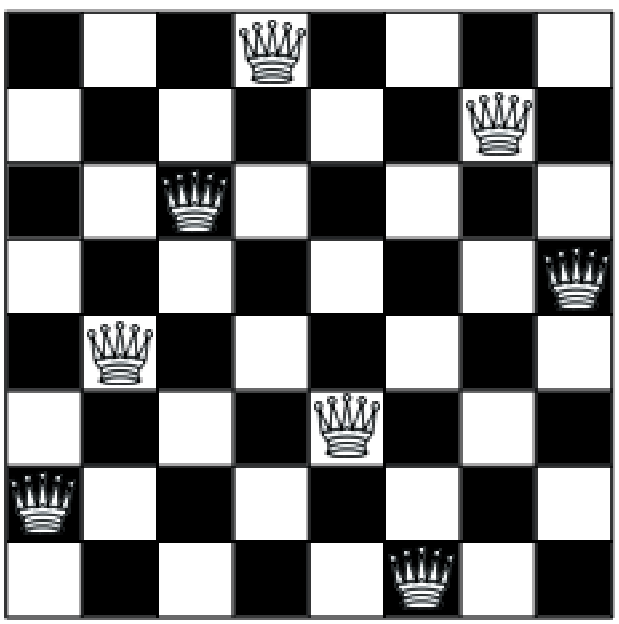
Using this equation we find that there are 204 squares on a chess board.

2. **Queens puzzle**
   In chess, the queen can move any distance in any direction as shown:



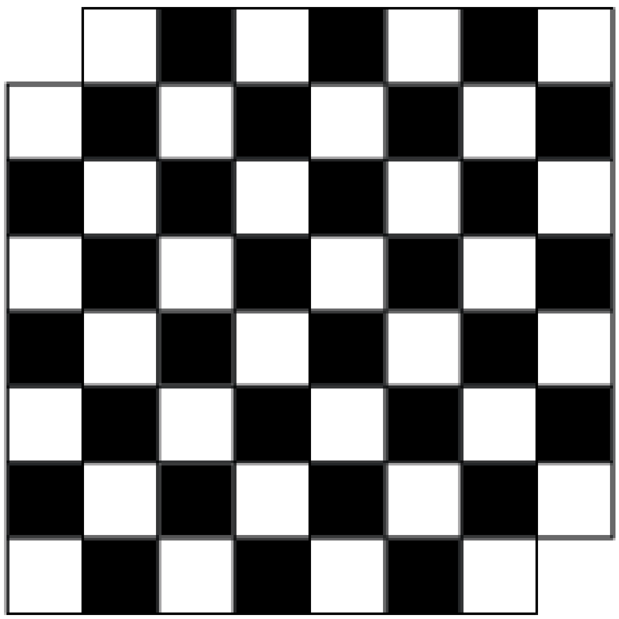
   In the queen's puzzle, the aim is to place 8 queens on an 8×8 chessboard so that no queen is threatening any other.

   Solution: Excluding symmetry there are 12 solutions to the Queens Puzzle. Here is one:



3. **Dominoes on a Chessboard**

   Suppose we have 32 dominoes, each which is the area of 2 squares on a chess board. It is easy to tile the chessboard with the 32 dominoes. Now suppose we take out two opposite corners of the chess board and take away 1 domino. Find a tiling of the remainder of the chess board with the 31 dominoes.

   Solution: This puzzle is impossible to complete and has a really simple, smart proof.

   We have taken 2 black squares out of the chessboard, so there are 30 black squares and 32 white squares left on the board. However each domino occupies 1 black square and 1 white square. So, after putting down 30 dominoes there will be 2 white squares left and we can not put down the final one. And so, it is impossible.

Take home problem

Place the numbers 1 to 8 in each of the circle such that no 2 consecutive numbers are in circles joined by lines.