Coláiste An Spiorad Naoimh Maths Circle
Lesson 1

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Warm-up Calculation

Take any 3 digit number, say 123. Write this number down twice, to get $123,123$. Now divide this by 7. Do you have any remainder? Now divide by 11. Do you have any remainder? Now divide by 13. You should have the same number you started with. Can you explain how this works?

1. ## Cupside Down

   This puzzle starts with 12 cups all upside down. The object is to turn all the cups up, but you must flip exactly 5 cups at a time. Each student can attempt the puzzle and the winner is the person that completes the challenge in the fewest moves.

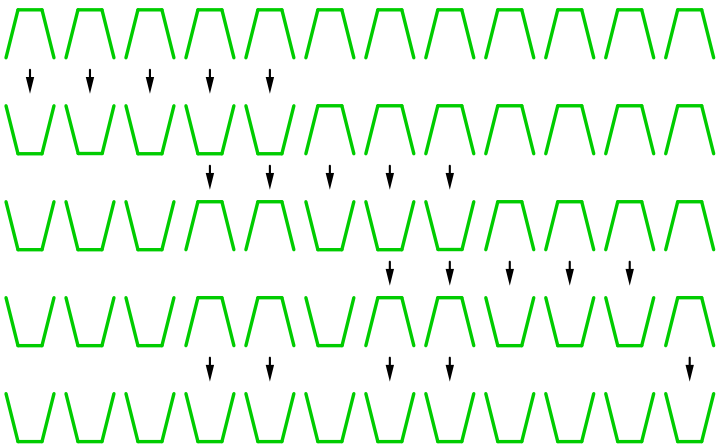
   Now start with 11 cups and this time, each move you must flip exactly 4 cups. Can the puzzle still be solved?

Solution

This is impossible and the solution is due to parity. We start with an even number(zero) of upright cups and we aim to turn an odd number(eleven) upright. However, each turn we are flipping an even number of cups. Whether we flip an even number of upright and upside down cups or an odd number of each, we still get a net change of an even number of cups flipped. As the sum of even numbers is always even, this puzzle is impossible.

We can use this property to answer a natural question about the first puzzle- what is the minimum number of moves needed to solve it. Because we need to flip an even number(twelve) of cups and on each move we flip an odd number(five), so we need to make an even number of moves. Below is a solution in 4 moves. After 2 moves the most we can flip is 10 cups, so 4 is the minimum number of moves needed to solve

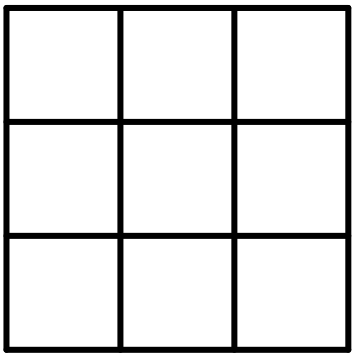
the puzzle.



2. # Magic Squares and Magic Circles

Magic Square

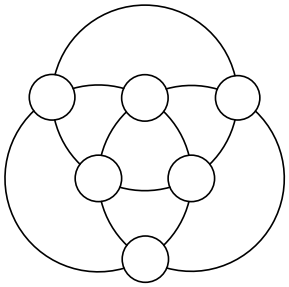
The object of a Magic square is to arrange the numbers 1-9 in a 3×3 box, so that the sum of the numbers in each row, column and diagonal add up to same number.



Student's should be encouraged to systematically find a solution as opposed to just trial and error.

- What is the sum of the rows/columns?
- Which number should go in the middle?
- Which numbers should go in the corners?

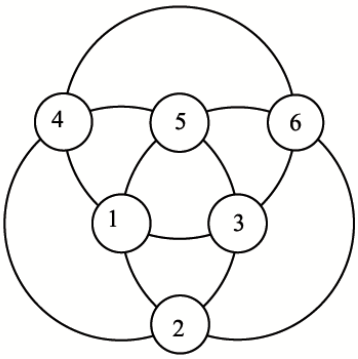
Magic Circles



Magic Circles is a puzzle, similar to Magic Squares- the object is to fill the numbers 1-6 in the small circles so that the numbers that lie on each of the big circles add up to the same number.

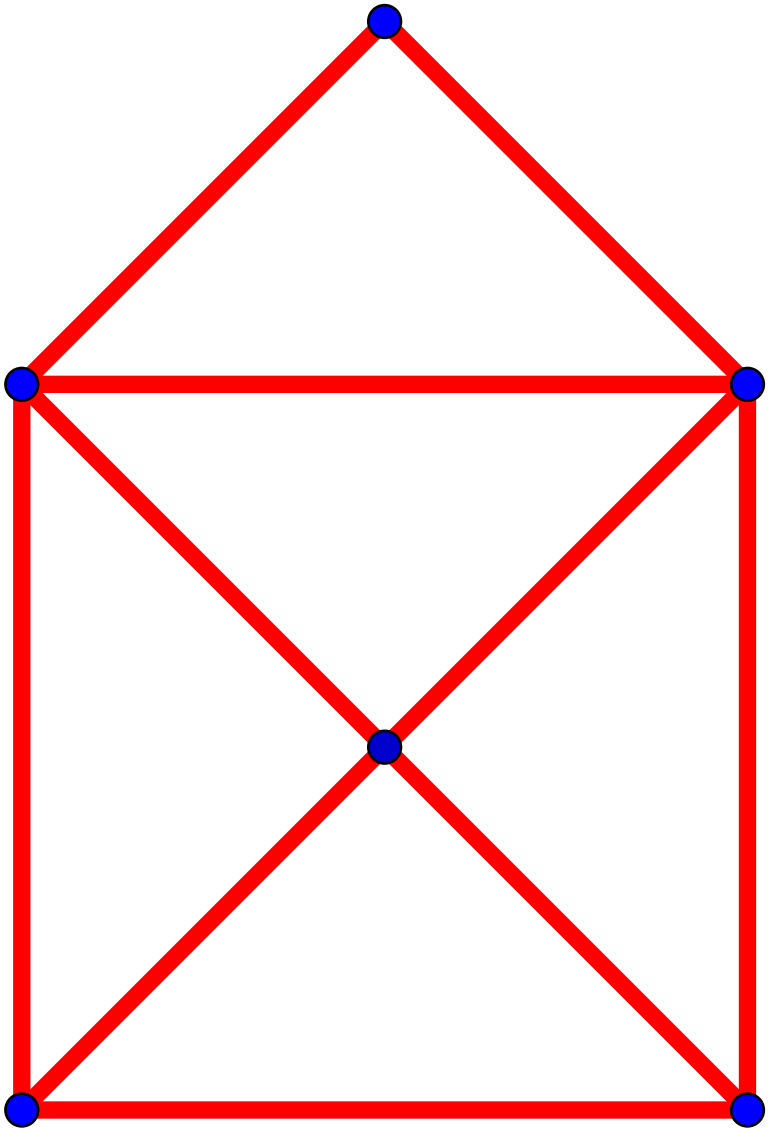
Solutions

4	3	8
9	5	1
2	7	6



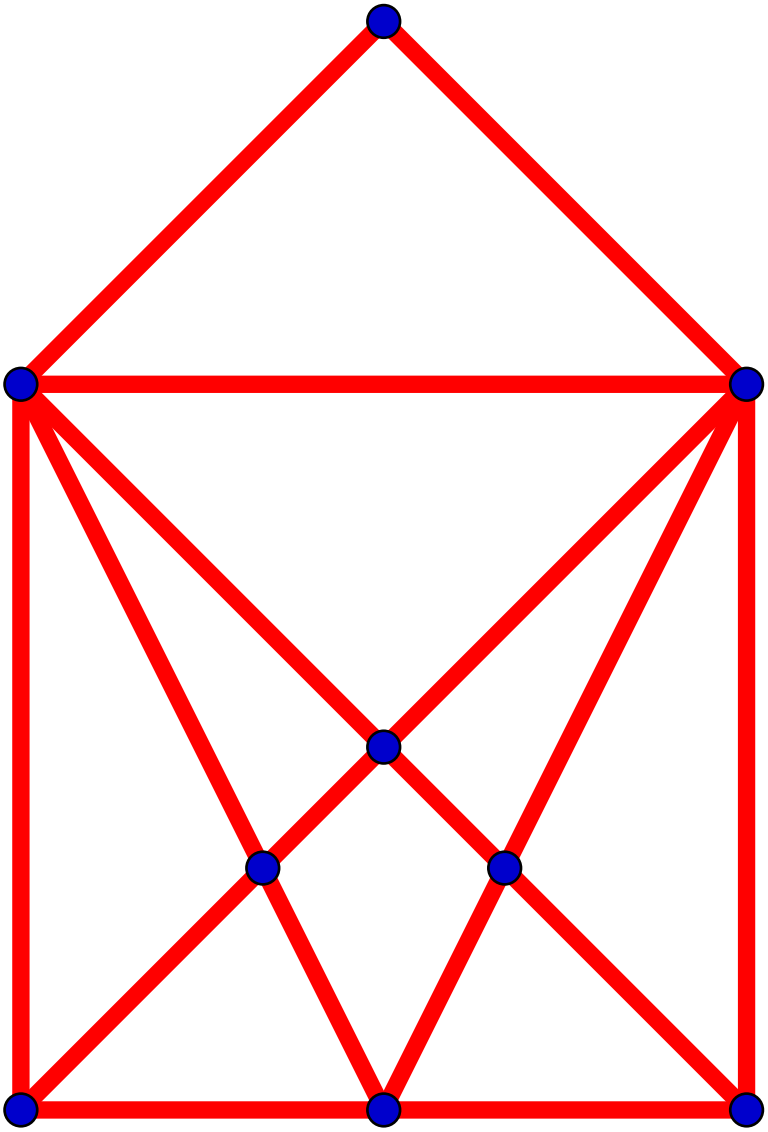
3. Eulerian Graphs

Shown Below is the map of the roads connecting a number of towns. The blue dots are the towns and the red lines are the roads.



Create a path that travels over each road, once and only once.

Now, new towns have been developed and more roads had to be built:



Can you find a similar path in the new network of roads?

Maths behind the puzzle

(a) We look at each of the towns and count the number of roads touching it.

(b) If we have an even number then for every time we drive into the town, there will be a road for us to leave on.

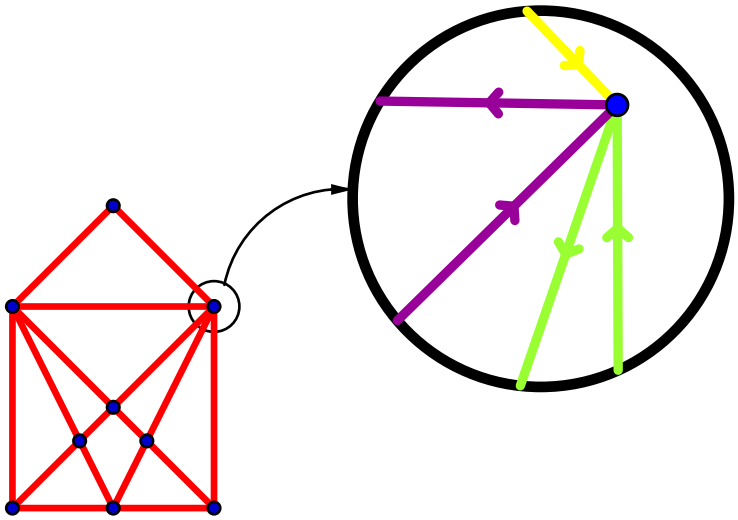


Figure 1: if we enter and exit by the green and purple roads first, when we then enter by the yellow road, there is no road left for us to exit by.

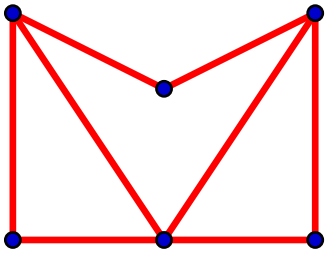
(c) If there is an odd number, then we must either start at that town but not finish there, or else finish there but not start there.

(d) So for there to be a possible path, there must either be no towns with an odd number of roads, in which case we start and finish at the same town, or else exactly two towns with an odd number of roads, in which case we start at one and end at the other.

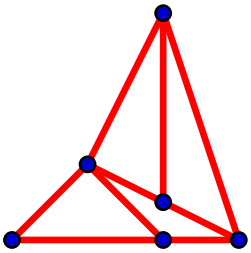
Graphs that have this property are called Eulerian graphs, named after Leonhard Euler, who discovered them.

Further Questions

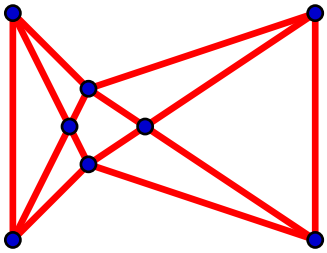
Which of the following are Eulerian graphs?



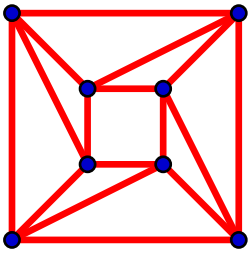
(a)



(b)



(c)



(d)

Take home problem

When Carl Friederich Gauss was 7 years old he had a lazy maths teacher! One day the teacher didn't feel like teaching at all, and to keep the class busy he told them to add up all the numbers from 1 to 100. In his head, Gauss thought of a very smart way of doing this and was able to shout out the answer in less than a minute! How did he do it?