

Coláiste An Spioraid Naoimh Maths Circle  
Lesson 15

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**Last Weeks Take Home Problem**

You buy 100 Kg of potatoes. They are 99% water. After a week in the sun their water content is 98%. How much do they now weigh?



**Solution:** 99% is water  $\Rightarrow$  1% is non water. so 1 Kg is non water. After a week in the sun, the water content is 98%, so the non water content makes up 2%. However, the non water content hasn't changed- it is still 1 Kg. So, now 2% = 1 Kg,  $\Rightarrow$  100% = 50 Kg.

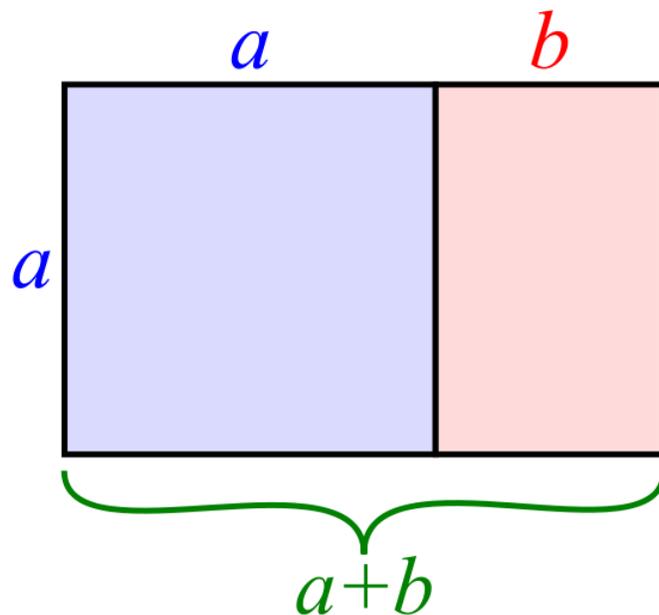
**Ans:** 50 Kg



## 1. Golden Ratio

- Pick any 2 numbers.
- Write the smaller one on the first line of a page and the larger one below it.
- Add the 2 together and write the sum on the third line.
- Now on each line write the sum of the numbers on the two lines above it, until you get to the bottom of the page.
- Now divide the last number by the one immediately before it. You should get 1.61803398875...

This is a very important number known as the golden ratio (represented by  $\phi$ ). Two quantities are in the golden ratio if the ratio of the sum of the quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one.



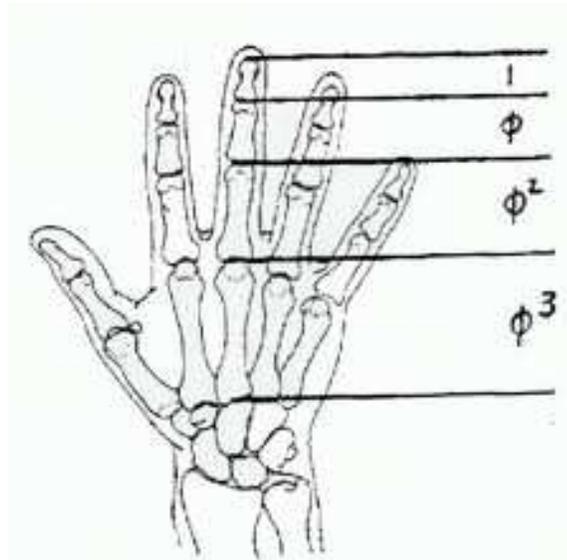
The rectangle above is known as the golden rectangle if the lengths of its sides are in the golden ratio to each other. i.e. if  $\frac{a+b}{a} = \frac{a}{b}$

Many artists and architects have proportioned their works to approximate the golden ratio, especially in the form of the golden rectangle, in which the ratio of the longer side to the shorter is the golden ratio, believing this proportion to be aesthetically pleasing. The golden ratio also pops up in many aspects of nature.

## Fibonacci Calipers



A Fibonacci Calipers is a tool for measuring the golden ratio. As you spread the two exterior legs, the middle leg will move accordingly, maintaining the golden ratio in the distances between the three legs. The golden ratio can be found in the bones in your fingers, as shown below.



Use the Fibonacci Calipers on your fingers to see how it works.

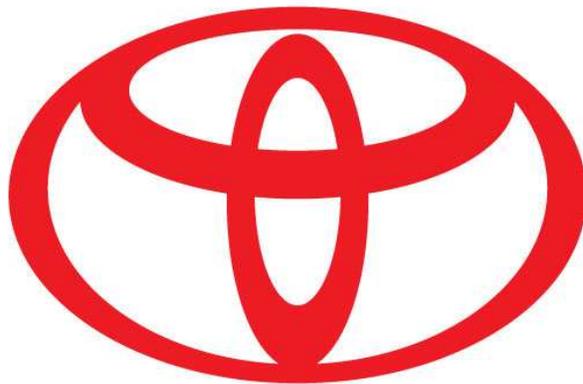
Now that we know how to use the Fibonacci Callipers, use it to find golden ratios/rectangles in the following:



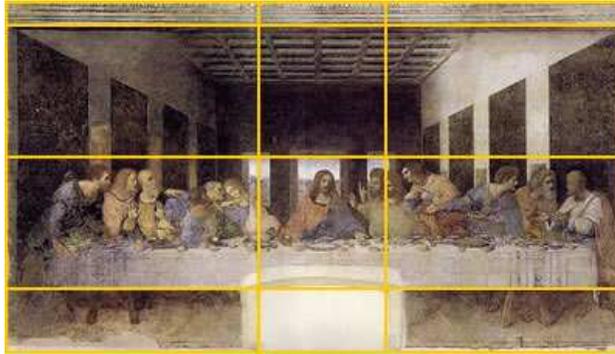
(a) The Last Supper



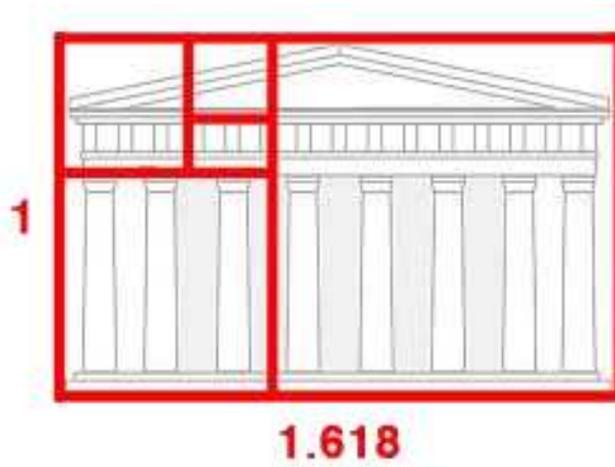
(b) The Parthenon



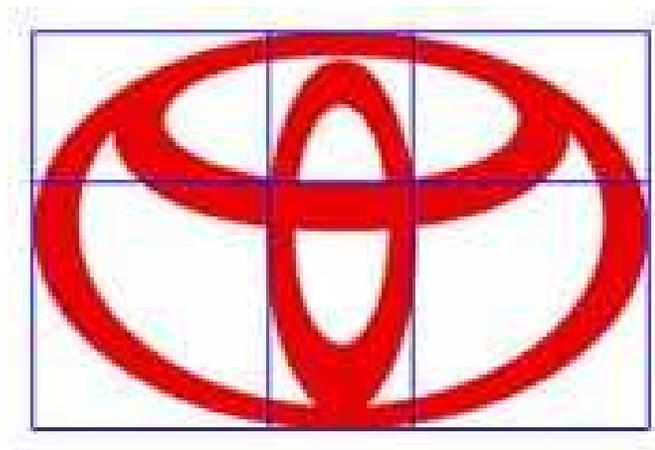
(c) Toyota Logo

**Solution:**

(d) The Last Supper



(e) The Parthenon

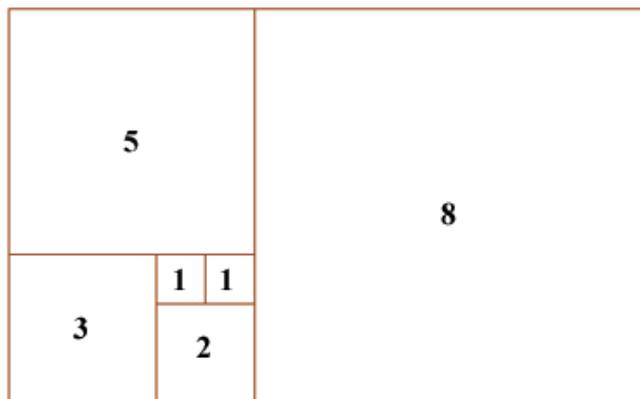


(f) Toyota Logo

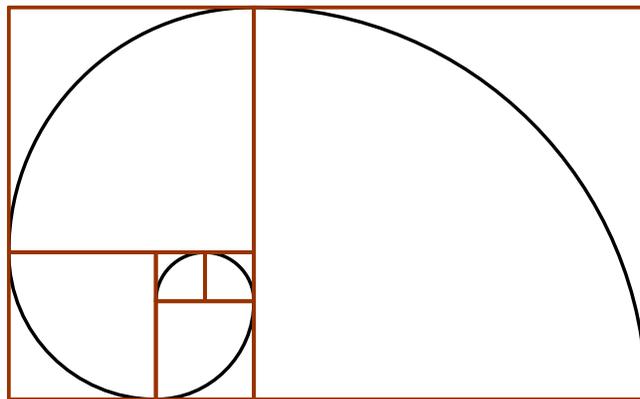
## 2. Fibonacci Spirals

We are going to learn how to draw a “Fibonacci Spiral”

- Start by drawing a  $1 \times 1$  square on the graph paper.
- Now draw another  $1 \times 1$  square, joined onto this one (to the right), to make a rectangle.
- At each step we add a square to our rectangle, spiralling each new square around the rectangle,  $\rightarrow \downarrow \leftarrow \uparrow \rightarrow \downarrow \leftarrow \uparrow \dots$
- Continue this until no more squares can fit in your page.
- Inside each square, write it's length.
- What do you notice???



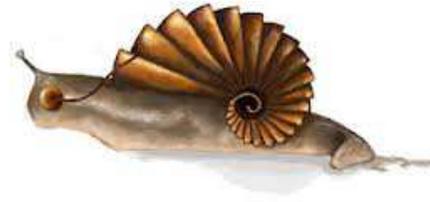
The lengths of each of the squares will be the Fibonacci Numbers. This is called the “Fibonacci Rectangle”. Use your compass to draw circular arcs inside the squares to get a “Fibonacci Spiral”



The Fibonacci Spiral appears in many areas of nature:



(g) The Nautilus Shell



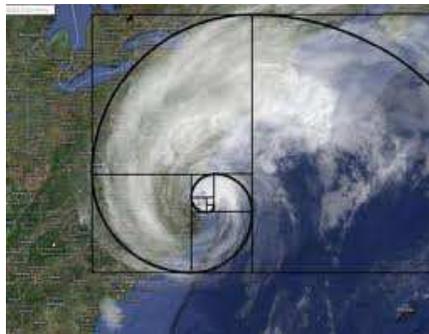
(h) A Snails Shell



(i) The Human Ear



(j) A Hen's Egg



(k) Hurricanes



(l) The Milky Way

## Take Home Problem



You are on a tour of Fibonacci's house- the elevator is broken and so you must take the Fibonacci stairs. The rules are, you can move up either 1 or 2 steps at a time but no more! If you start on step 1 how many ways can you get to step 10?