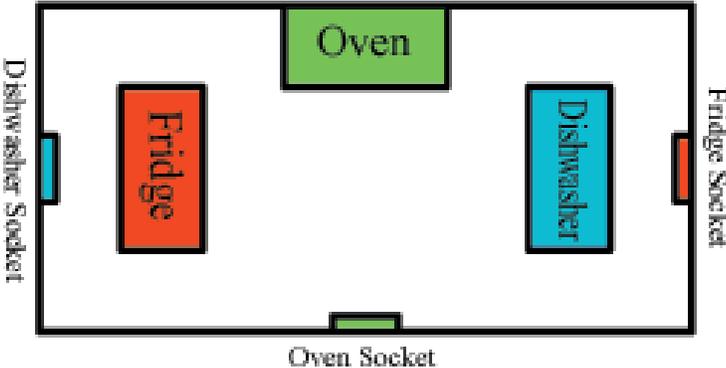# Coláiste An Spioraid Naoimh Maths Circle
Lesson 14

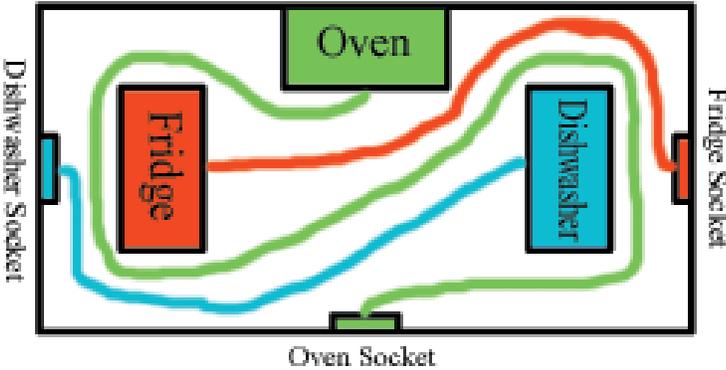Robert Linehan
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Last Weeks Take Home Problem

Can you plug each of the appliances into their appropriate sockets such that none of the plug cords cross over.

Solution:

Mathemagical Card tricks

1. ## "Will The Cards Match?"

   http://www.youtube.com/watch?v=98v3zOXentA

   This card trick uses just 10 cards- the ace through to the 5 of hearts and of spades. The performer splits the cards into two decks- one with all the hearts and one with all the spades. The cards are turned face down and the performer starts to spell out the sentence, "Will the cards match", each time moving the top card from one of the decks to the bottom. The spectator can say "switch" at any time and the performer will switch to the other deck. At the end of each word, the top card on each deck are placed together to one side, and the procedure continues with the remaining decks. At the end of the sentence, you will have 5 pairs of cards- the cards are turned over, and all the cards are matched. i.e. the two aces are together, etc.

   How To Perform:

   The trick is mostly self working.

   - Arrange the ace through to the five of spades, and the ace through to five of hearts and place them on top of each other.

   - Fan the ten cards face up, to show the spectator.

   - Now turn the cards over and deal the top 5 cards off the deck to create 2 decks. (This reverses the order of one of the decks)

   - The rest of the trick is self working!

Why does this trick work???

Solution:
The best way to understand how this trick works is to perform it with
the cards face up, but let's try to explain it mathematically:

Let's look at the first word, "Will". There are 5 cards in each deck-
let's number them $1, 2, 3, 4, 5$. Deck A starts on card 1 and counts up
to card 5 and deck B starts on card 5 and counts down to card 1. There
are 4 letters in "Will"- suppose we move x cards in deck A- that means
the top card will be card $1+x$. So the rest of the cards will be moved in
deck B- which will be $4-x$, so it will be on card $5-(4-x) = $ card $1+x$.

The same works with the second word, "The". Let's look at the third
word "Cards" There are 5 letters and just 3 cards left- $1, 2$ and 3. Deck
A starts with card 1 on top, and deck B starts with card 3 on top.
Suppose we move x cards in deck A- it now has card $1 + x$ on top. we
then move $5 - x$ cards in deck B- so card $3 - (5 - x) = $ card $x - 2$ is
on top. There are 3 cards left in the deck so when we take 2 from x,
we get back to card $x + 1$. This may be better understood from the
table:

$\cdots$	$x-3$	$x-2$	$x-1$	x	$x+1$	$x+2$	$x+3$	$x+4$	$x+5$	$\cdots$
$\cdots$	x	$x+1$	$x+2$	x	$x+1$	$x+2$	x	$x+1$	$x+2$	$\cdots$

In short, the reason that this trick works is that the number of letters
in each word is 1 less than a multiple of the number of cards left.

2. To Your Heart's Content

This trick just uses 20 cards- the ten through to Ace of each suit. The magician gives the spectator a number of chances to (seemingly) randomise the cards, each time "To Your Heart's Content". The cards are then spread to reveal the five hearts face up and all other cards face down.

How to Perform:

- Hand the spectator the 20 cards and tell them to shuffle them "to your heart's content"

- Go through the cards 2 at a time, sometimes having them 2 cards facing up, sometimes having them face to face and sometimes having them back to back, and ask the spectator does he want them flipped up or down. (secretly, if there's a heart and a non heart you put them both facing up with the heart on top, 2 hearts go back to back and 2 non hearts go face to face)

- Ask the spectator are they flipped "to your heart's content". Just in case, tell them you will give them one more chance- this time deal them in piles of four and ask if they want the pile flipped up or down.

- Now deal them into 4 rows of 5. For each row you can ask them if they want it dealt left to right or right to left.

- Now ask the spectator to select an edge of the grid- you fold in this edge like you are folding a sheet. Continue to do this until the cards are all in one pile.

- Confirm with the spectator that the cards have been shuffled, flipped and folded all to their heart's content and then spread the cards to reveal the 5 hearts face up.

Why does this trick work?

Solution:
This trick is based on parity. Let's look at the grid formed at the end:
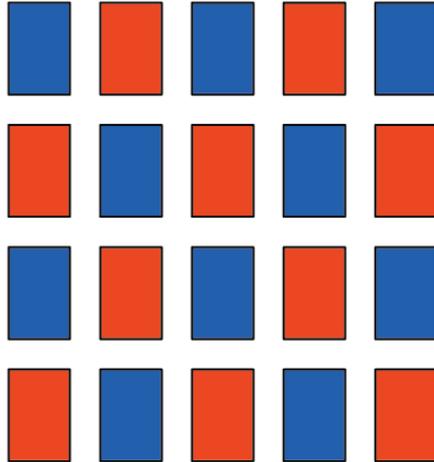
- Each of the blue cards are in an odd position and each of the red cards are in an even position. Due to the symmetry of the grid, this is independent of whether each row is dealt right to left or left to right.

- Each fold sends the cards to a position of the opposite colour. So if a card is face up on a blue position it will be face down on a red position and vice versa.

- The way you arrange the cards in the flipping stage of the trick ensures that the hearts will be facing up in an odd position and down in an even position and that the non hearts will be the opposite.

- In the second flipping stage the spectator is given the option to flip 4 cards at once- this does not effect the set up as it will switch which way each card is facing but also switch the parity of the card.

- And so, when you fold all the cards into one pile, the hearts will face up and the non hearts will face down or vice versa in which case you flip the deck over before spreading them.

3. **The Final 3**

http://www.youtube.com/watch?v=oLjEulT6ssM

3 spectators choose a card each- the cards are returned to the deck and the deck is dealt onto the table with alternating cards being turned up. This is repeated with the cards turned down until there are just 3 cards left. These three cards are the spectators cards.

How to Perform:

- Have the three spectators select a card each.
- Tell them to show each other and memorize each others cards. Meanwhile you divide the deck into three piles- the first has 10 cards, the second and third both have 15 cards. (You should have 9 cards left in your hand)
- Tell a spectator to put his card on top of the first pile and cut as many cards as he wants from pile 2 onto pile 1.
- Then the next spectator put's his card on top of pile 2 and cut as many cards as he wants from pile 3 onto pile 2.
- The last spectator puts his card on top of pile 3.
- Now put 5 of the remaining cards on top of pile 3. Put pile three on top of pile 2, this on top of pile 1, and all this on top of the remaining 4 cards.
- Now start dealing the cards into two piles, turning over each card in one of the piles (the one that you deal the first card into). Tell the spectators that whoever sees their card first wins a prize.
- Repeat this with the face down deck, until there are just 3 cards left. Turn over the three cards to reveal the spectators cards.

Why does this trick work?

Solution:

Firstly, It doesn't matter which way the spectators cut the piles- when you stack the piles together, you will be moving the rest of the pile, so regardless of which way they cut the piles, there will be 15 cards in between each of their cards.

Suppose we number the 52 cards in the deck. After we deal them as described above until there are 3 cards left, the cards will be card 6, card 22 and card 38 (or cards $6 + 16n, n \in \{0, 1, 2\}$). We know that the first card will be in position 6 as we put 5 cards on top of it. Because there are 15 cards in piles 2 and 3, there will be 15 cards between each of the spectators, and so the second and third cards will be in positions 22 and 38.

Take Home Problem

You buy 100 Kg of potatoes. They are 99% water. After a week in the sun their water content is 98%. How much do they now weigh?