Coláiste An Spioraid Naoimh Maths Circle
Lesson 13

Robert Linehan
Lorcán Mac An tSaoir
Peter Fagan
Denis O Sullivan

February 22, 2013

Last Weeks Take Home Problem

An old rich Arab had 17 Camels. Before he died he had instructed that his camels be divided between his three sons- the oldest was to receive half of them, the middle son was to receive one third and the youngest was to receive one ninth. After his death the three sons were confused- how could they have half, a third or a ninth of 17 without cutting any of the camels to pieces??

Then a stranger passes on a camel, and the oldest son had a great idea- they borrow the strangers camel, giving them 18. The oldest now takes 9 of them, the middle son gets 6 and the youngest gets 2. $9 + 6 + 2 = 17$, so they can give the stranger back his camel. How is it, that this worked out so conveniently?

Solution:
The trick here is that $\frac{1}{2} + \frac{1}{3} + \frac{1}{9} = \frac{17}{18}$, so in order to divide up all 17 camels, they needed to include an extra camel.
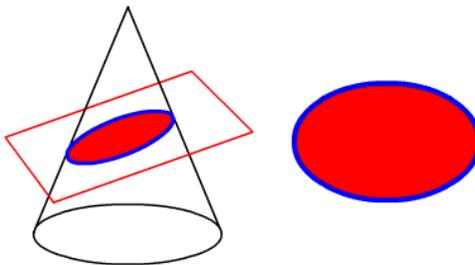
Conic Sections
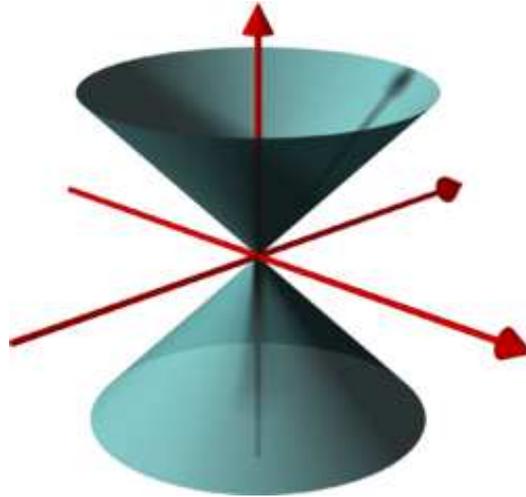
Since about 200 B.C. Mathematicians have been studying conic sections-
the curve obtained when a plane intersects a right circular conical surface.
What is a conical surface? Basically, it's two infinitely long cones, whose
points are glued together as shown in the picture, above.

If the intersecting plane is horizontal, then the conic section will be a circle.
What kind of curves do you think you will get if you tilt the intersecting
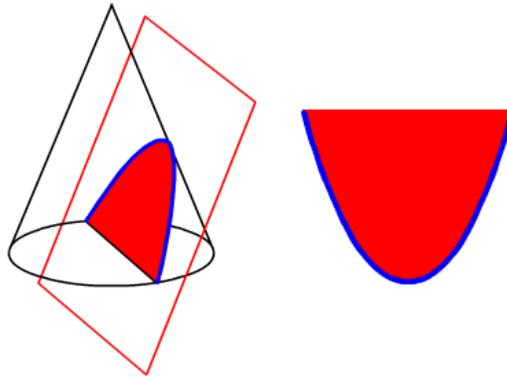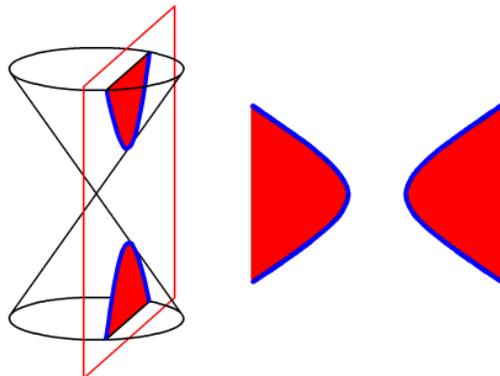plane? Sketch any curves that you think you will obtain.

Solution:

(a) Ellipse

If the slope of the plane is less than the slope of the edge of the cone,
then the conic section will be an "ellipse".

(b) Parabola

If the slope of the plane is equal to the slope of the edge of the cone, then the conic section will be a "parabola".

(c) Hyperbola

If the slope of the plane is greater than the slope of the edge of the cone, then the conic section will be a "hyperbola". Notice that the plane cuts both "nappes" of the conical surface, giving the hyperbola 2 branches.

1. **Drawing Ellipses**
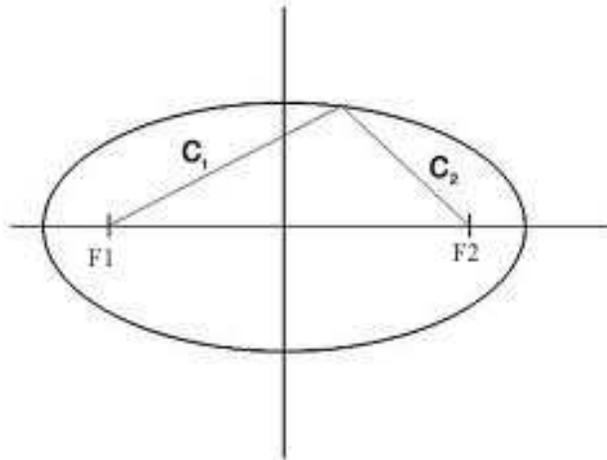
So now, it's about time we construct some conic sections. In a circle, every point on the curve is a constant distance from a certain point- the center. In an ellipse, there are two special points, called the foci (a focus and another focus) that act in a similar fashion. If we call the foci points f_1 and f_2 and let c_1 and c_2 be the distances between a point on the curve and f_1 and f_2 respectively, then $c_1 + c_2$ will be a constant. We can use this fact to draw an ellipse.
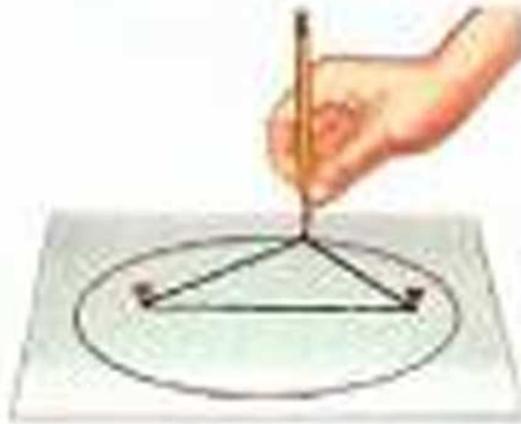

- Put your page on top of the corkboard.
- Put the pins where you want your foci to be.
- Make a loop from the string and place it around the pins.
- Place a pencil inside the loop. Pull the loop tight and move your pencil around the pins to trace out an ellipse

Uses of Ellipses

(a) **Elliptic Orbits**

In the 17th Century, John Kepler discovered that orbits at which the planets travel around the sun are ellipses with the sun at a focus.
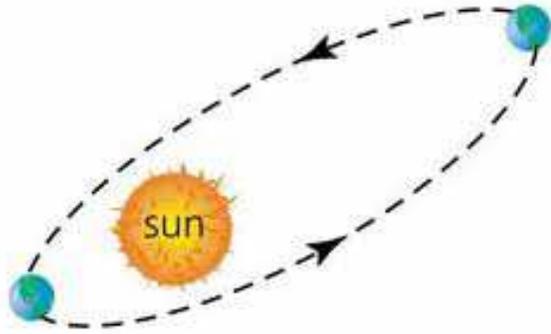

(b) **Whisper Chamber**

A whisper chamber is a large elliptical room. One person stands at one focus and another stands at the other. The two people can talk such that people in between them wont be able to hear them but they will be able to hear each other.

2. **Drawing Parabolas**

Just as with an ellipse, there are many different ways of drawing parabolas. One of the coolest, just uses straight lines! Start with two lines of equal length, meeting at a point. Put a number of equally spaced points on each line:
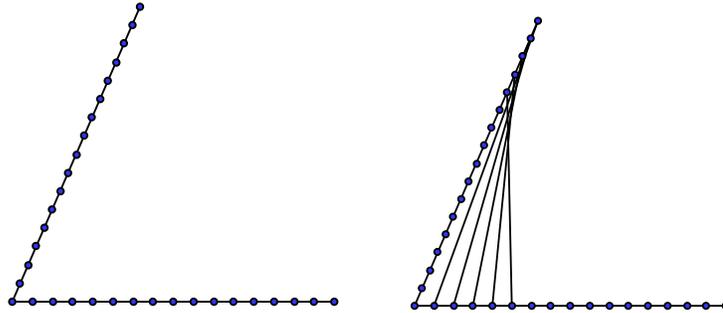
Draw a straight line between the farthest point on one line and the nearest point on the other. Now draw a line between the second farthest and the second nearest, and then the third farthest and the third nearest, and so on. Continue with this pattern until all the lines are drawn and prepare for your mind to be blown!!
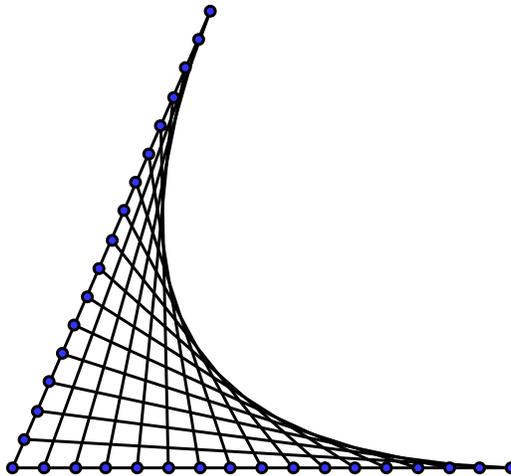
All the straight lines make up the "envelope" of a parabola, meaning that they all lie tangential to it.

Uses of the Parabola

(i) **Projectiles**

The best known instance of the parabola in the "real world" is the trajectory of a projectile. This was discovered by Galileo in the early 17th century.


So every time you fire an angry bird you will see a parabola! Also every time you throw or kick a ball, throw darts, spray water from a hose, etc, the trajectory will be a parabola.
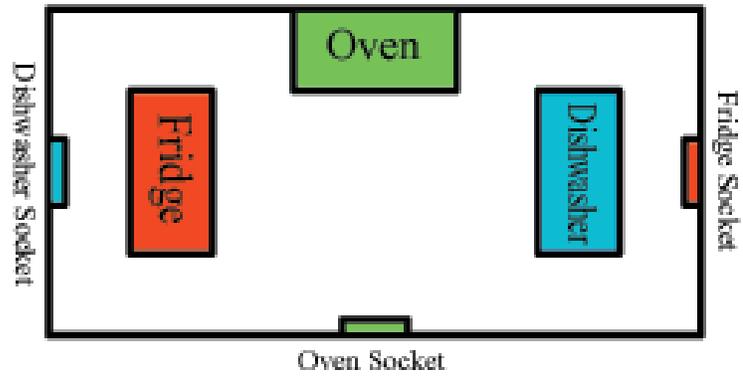
(ii) **Car headlights**

The headlights on your car are based on having a parabolic mirror around the light bulb. The same principal is used for satellite dishes, hearing aids, etc.

(iii) **Suspension Bridges**

Approximations of parabolas are also found in the shape of the main cables on a simple suspension bridge.
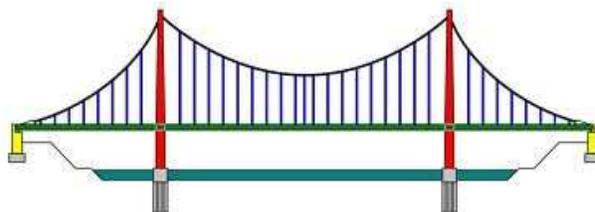
Take Home Problem

Can you plug each of the appliances into their appropriate sockets such that none of the plug cords cross over.