Coláiste An Spioraid Naoimh Maths Circle
Lesson 12

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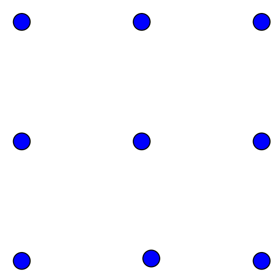
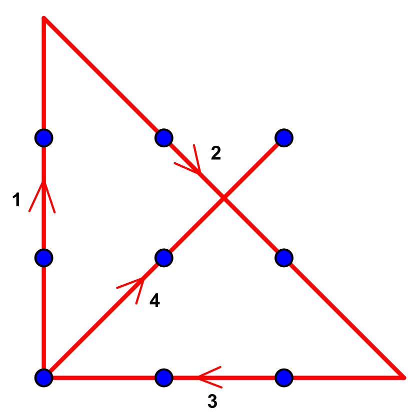
Last Week's Take-Home Problem



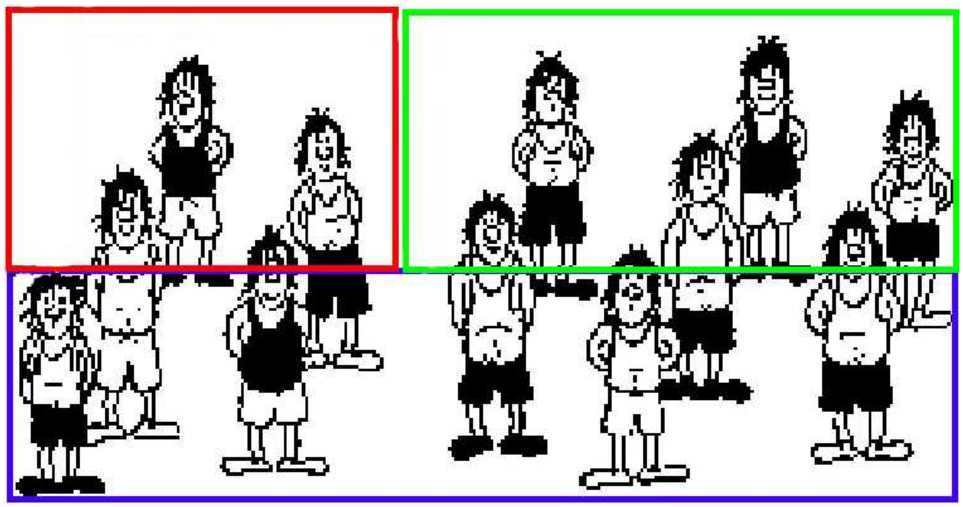
The object of this puzzle is to link each of the 9 dots using 4 straight lines without lifting your pen from the paper and without retracing any of the lines.

Solution:

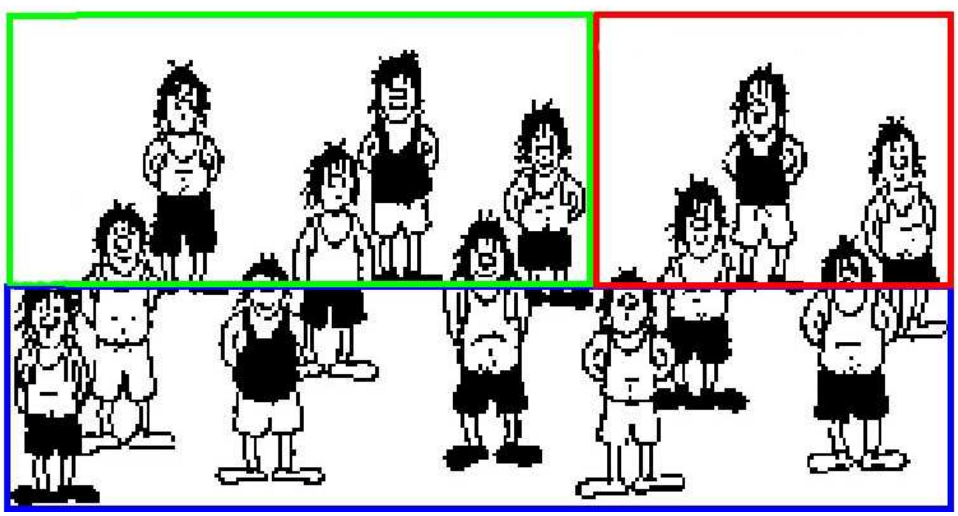
The trick here is to think "outside the box":



1. **12 or 13 man illusion**
   This is a really amazing and confusing optical illusion. You are presented with a picture of some cartoon men, which is cut into 3 pieces- 1 long base, and 2 shorter pieces to fit on top of it. Count the men in the picture. Now switch the two short pieces of the picture and count the men again. What do you notice?



(a) Frame a

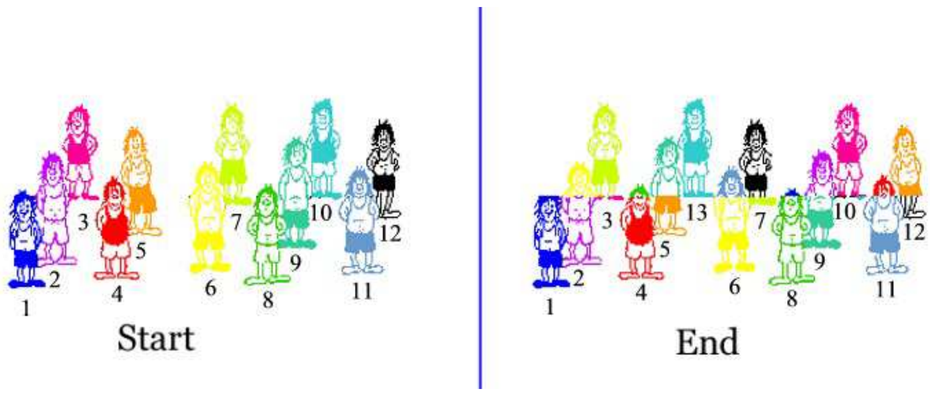


(b) Frame b

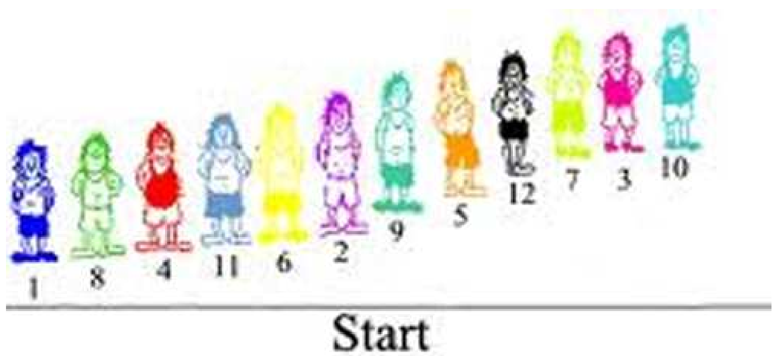
In frame a there are 12 men, but in frame b there are 13 men. Play with the picture for a few minutes and try to explain how this can be.

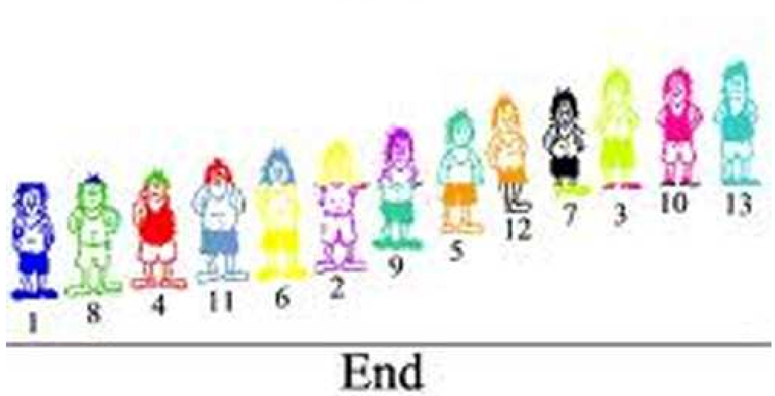
Solution:

Each of the men can be broken into 13 parts from head to toe. As the frames are moved, each new person is missing one of the sections, so we get 13 people, each $\frac{12}{13}$ of their original height. How do we not notice this??? The low resolution of the picture helps the illusionist to trick us. It is easier for us to see how this works if we colour the people:



We can see now where each of the men goes when the frames move and compare their new top section to their old one. It is even easier to see how the illusion works if we now rearrange the men in order of how high they are positioned on the page:

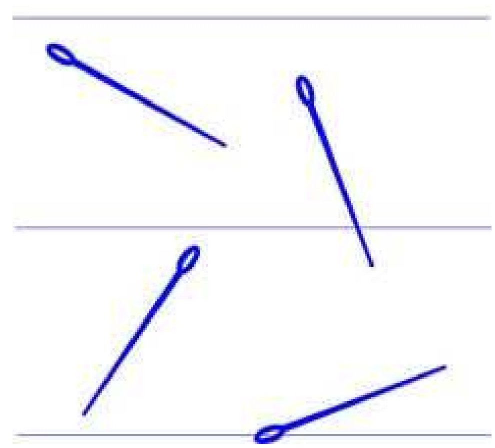




2. Estimating π using Buffon's Needle

The ratio of a circle's circumference to it's diameter, i.e. π has been studied for over 4000 years. All throughout history, Mathematicians developed many different methods of estimating the value of π. In my opinion, the most interesting method uses geometric probability theory. In 1777 Georges-Louis Leclerc, i.e. the Count of Buffon posed a problem:

Suppose we have a needle of length l, and we drop it onto a plane ruled with parallel lines spaced distance l apart. What is the probability that the needed will rest across one of the lines?



The answer to this problem is $\frac{2}{\pi}$. This fact can be used to estimate π. In pairs, take a needle and a lined page. One student should toss the needle onto the page randomly. The other should write a "1" if the needle crosses a line and a "0" if it doesn't. This should be repeated for 5-10 minutes, to collect as much data as possible. When you have all your data, count how many "1"s and "0"s you have. We can then estimate π:

$$\pi \approx \frac{2 \times (\#1 + \#0)}{\#1}$$

The more data that you use, the more accurate your results should be- In 1901 Mario Lazzarini, an Italian Mathematician performed the experiment, tossing a needle 3408 times and got an error less than 0.0000003

Take Home Problem

An old rich Arab had 17 Camels. Before he died he had instructed that his camels be divided between his three sons- the oldest was to receive half of them, the middle son was to receive one third and the youngest was to receive one ninth. After his death the three sons were confused- how could they have half, a third or a ninth of 17 without cutting any of the camels to pieces??

Then a stranger passes on a camel, and the oldest son had a great idea- they borrow the strangers camel, giving them 18. The oldest now takes 9 of them, the middle son gets 6 and the youngest gets 2. $9 + 6 + 2 = 17$, so they can give the stranger back his camel. How is it, that this worked out so conveniently?