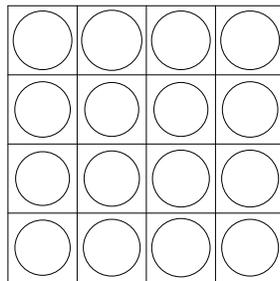


Coláiste An Spioraid Naoimh Maths Circle  
Lesson 11

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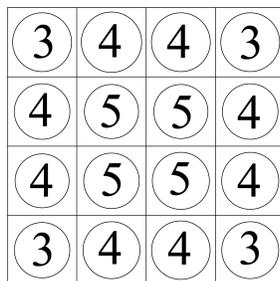
January 25, 2013

**Last Week's Take Home Problem**



Suppose we pressed each bulb in the lights out grid, one at a time- recall that each time we press a bulb it changes the state of that bulb, the bulbs directly above/below it and the bulbs to the right/left of it- what bulbs would be on/off at the end? Is the answer the same no matter what order you press the bulbs?

**Solution:**

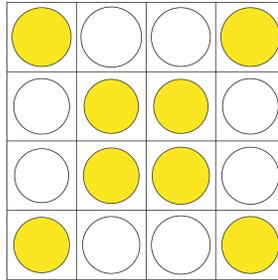


Last week, we used the above figure to show how many bulbs changed state when each bulb was switched. We get the same figure if we ask for each

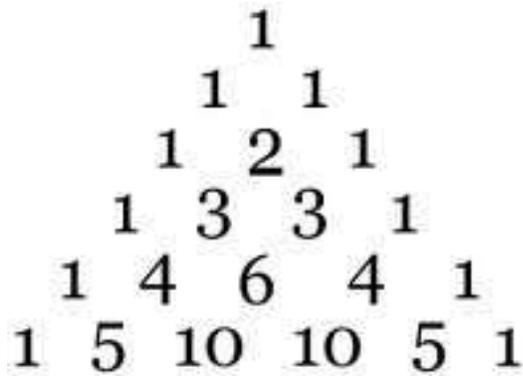
bulb, how many bulbs are there that when switched will change its state. So in other words, the figure shows the amount of times that each bulb will change state if every bulb is switched, one at a time.

0	1	2	3	4	5	6	...
off	on	off	on	off	on	off	...

After an odd number of changes a bulb will be on and after an even number it will be off, so the bulbs that will be on are as follows:



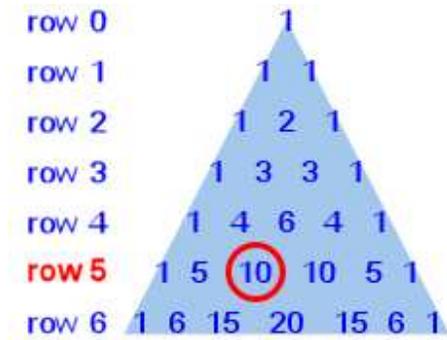
## Pascal's Triangle



This pattern is named after the French mathematician Blaise Pascal (1623-62), who brought the triangle to the attention of Western mathematicians. Each number in the triangle is the sum of the pair of numbers directly above it (to the above left and above right). Above, just the first six rows of the triangle is shown, but there is no limit to the amount of rows in Pascal's Triangle.

### 1. Using Pascal's Triangle for Combinatorics

Suppose that we have a bowl of fruit with apples, oranges, bananas, grapes and strawberries and we want to choose 2 different pieces of fruit. How many different ways can we choose them? We can find the answer from Pascal's Triangle. There are 5 different fruits, and we want to choose 2, so we go to row 5 (counting the first row as row 0), and take the number from position 2 (counting the first position, "1" as position 0)

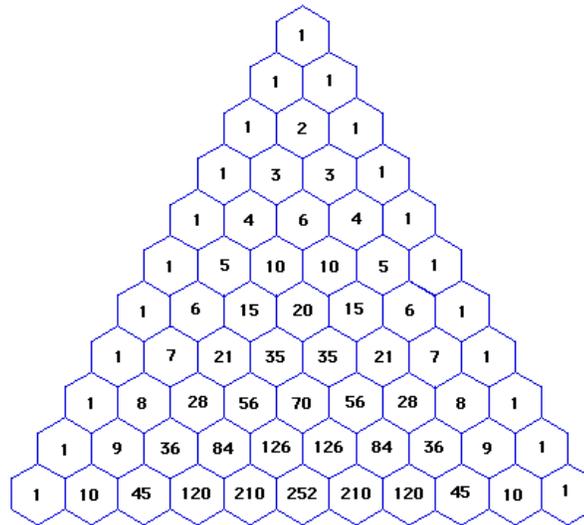


We can simply read the answer, "10" from the triangle.

Make a Pascal's triangle with 10 rows and use it to answer the following questions:

- Suppose a pizza shop has a choice of 10 toppings. How many ways can you make a pizza with exactly 4 toppings?
- If you have enough money to buy a pizza with up to 5 toppings, how many choices do you have?
- If half of the toppings are meat, what portion of the pizzas are vegetarian?

**Solution:**



- (a) Just reading from the triangle, Ans = 210.  
 (b) This is the number of ways to choose 0, 1, 2, 3, 4 or 5 toppings.  
 Reading these numbers from the triangle:

$$1 + 10 + 45 + 120 + 210 + 252 = 638$$

- (c) First we calculate the total amount of pizzas that can be made from a total of 5 toppings, reading from row 5:

$$1 + 5 + 10 + 10 + 5 + 1 = 32$$

Next, we calculate the total amount of pizzas that can be made from a total of 10 toppings, reading from row 10:

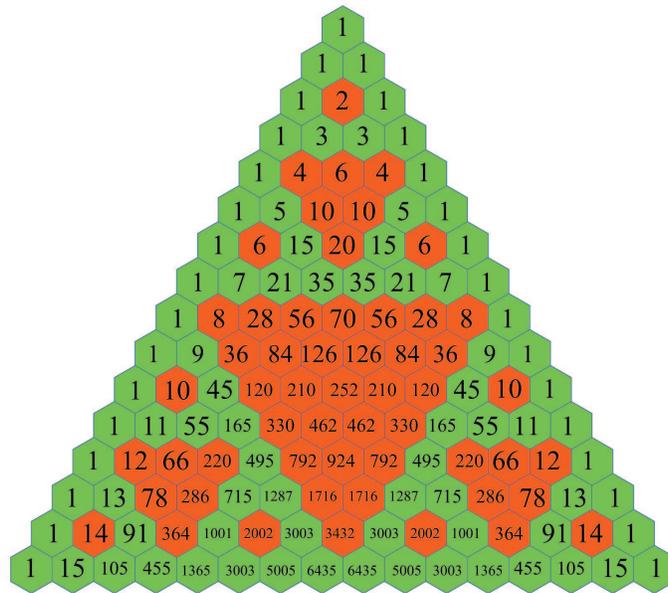
$$1 + 10 + 45 + 120 + 210 + 252 + 210 + 120 + 45 + 10 + 1 = 1024$$

Now, using both these numbers we see that the portion of pizzas that are vegetarian is  $\frac{32}{1024} = \frac{1}{32}$

## 2. Colouring Pascal's Triangle

Take a Pascal Triangle with 16 rows. Colour all the even numbers one colour and all the odd numbers another. What pattern do you get? Can you explain why it makes this pattern? If you make the triangle bigger, will this pattern continue?

**solution:**



The pattern resembles Sierpinski's Triangle, which we met last week. This is a result of:

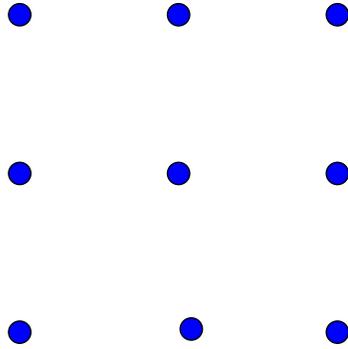
$$\text{even} + \text{even} = \text{even}$$

$$\text{even} + \text{odd} = \text{odd}$$

$$\text{odd} + \text{odd} = \text{even}$$

Every time there are  $n$  consecutive numbers of the same parity (odd or even) in a row, there will be exactly  $n - 1$  even numbers below them, in the next row. This explains why inverted red triangles are formed. For every red triangle, because the numbers just outside it are odd, and the numbers down along the outside of Pascal's triangle are odd (1), the same pattern is formed to the left and the right of the red triangle, as above it. This demonstrates the self similar property of the Sierpinski Triangle, and that the pattern will continue if you increase the number of rows of Pascal's Triangle that you looked at.

## Take Home Problem



The object of this puzzle is to link each of the 9 dots using 4 straight lines without lifting your pen from the paper and without retracing any of the lines.