Draft

Sketch of Lesson Plans on Fractions

1) Basic additions.

Most students would probably know how to add fractions correctly, but you'd like to make sure of it without getting into boring routine staff.

Example ① Amazing fractions:

Calculate:

a) $\frac{11}{9} + \frac{9}{11} =$

b) $\frac{101}{99} + \frac{99}{101} =$

c) $\frac{1001}{999} + \frac{999}{1001} =$

(Can you guess c) without calculating? Maybe: Use a calculator to check your answer).

Extensions: • Can you explain what is going on?
• Can you come up with more examples (of patterns) like this one?

Solution: $\frac{11}{9} + \frac{9}{11} = \frac{11 \cdot 11 + 9 \cdot 9}{9 \cdot 11} = \ldots = \frac{202}{99}$

$\frac{101}{99} + \frac{99}{101} = \ldots$ fill in $\ldots = \frac{20002}{9999}$

$\ldots$ fill in $\ldots$

General idea: $\frac{x+1}{x-1} + \frac{x-1}{x+1} = \frac{(x+1)\cdot(x+1) + (x-1)(x-1)}{(x-1)(x+1)}$

Expand $(x+1)\cdot(x+1) = x^2 + x + x + 1 = x^2 + 2x + 1$.
$(x-1)\cdot(x-1) = x^2 - x - x + 1 = x^2 - 2x + 1$.

Add: $\Rightarrow 2(x^2+2)$

2) Fractions and divisibility (factors of a number).

[Naturally this implies that fractions would come after divisibility. Or, this could be a motivation for working on divisibility?]

a) In this section maybe you can propose some sums of fractions with big denominators, but also large common factors... see how the students approach the summing problem. Most of them may just try to multiply the denominators. So you can show how much easier it is to use the l.c.m. of denominators.

Example: $\frac{31}{4620} - \frac{30}{4389} =$ (no calculators)

Find other examples of this type.... ?
(Arguably, calculator use makes this kind of techniques less spectacular... maybe when used in some algebraic expressions?

b) More on fractions and divisibility

Example① Find all whole numbers n such that

$\frac{n+5}{n-1}$ is a whole number.

Solution: This fraction is "top heavy" (? is this a commonly used term?) so it already has some whole number hidden into it. We divide to discover that hidden number:

$n+5 = n-1+6$

the whole number hidden there...

So $\frac{n+5}{n-1} = \frac{n-1}{n-1} + \frac{6}{n-1} = 1 + \frac{6}{n-1}$.

$n-1$ must be a factor of 6, so it could be $6, 3, 2, 1, -1, -2, -3, -6$ ......

Follow up questions: Same for $\frac{6n+1}{n-4}$ ... Other examples?

Should we dare $\frac{n^2}{n-2}$?....

c) More on... Can we write $\sqrt{2}$ as a fraction? (and the proof by contradiction here........)

3) Play with equal fractions (For applications in similar triangles. So a similar triangles lesson plan may be inserted in this area...)

Open discussion : Consider the fractions:

$$\frac{9}{15} \quad \text{and} \quad \frac{33}{55}.$$

What can you notice about them?

(They're equal, because $\frac{9}{15} = \frac{3 \cdot 3}{5 \cdot 3}$ ....)

What if we take the numbers on top and bottom of one of the functions, play with them after some rules, and then we do the same game for the second fraction: Will the results still be equal?

Example:
- subtract numerator and denominator
- replace the denominator by the result:

$15-9=6$	$55-33=22$
$\frac{9}{6}$	$\frac{33}{22}$

still equal.

Other examples of rules: Check $\frac{9+15}{15} = \frac{33+55}{55}$

....

$$\frac{9+15}{15-9} = \frac{33+55}{55-33}$$

- can you use products:

$$\frac{9 \cdot 15}{15} \overset{?}{=} \frac{33 \cdot 55}{55}$$

If yes why? If not why do you think it doesn't work?

Can you ~~formulate~~ conjecture? what works in general?

If $\frac{x}{y} = \frac{x'}{y'}$ then ..... $\frac{Ax+By}{Cx+Dy} = \frac{Ax'+By'}{Cx'+Dy'}$

~~Solution...... Proof.......~~

or possibly settle for simpler conclusions ($A, B = 1$ etc).

Can you explain why this happens ?

(1) Possible 1st explanation proposed by some students may refer to the case when x, y, x', y' are whole numbers and so $\frac{x}{y}$, $\frac{x'}{y'}$ have the same reduced form $\frac{u}{v}$. So $x = Mu$, $y = Mv$ .... fill in details...

(2) In this case, propose the situation when some of x, y, x', y' may not be whole numbers, but still $\frac{x}{y} = \frac{x'}{y'}$. (example $x = \sqrt{2}$) ?

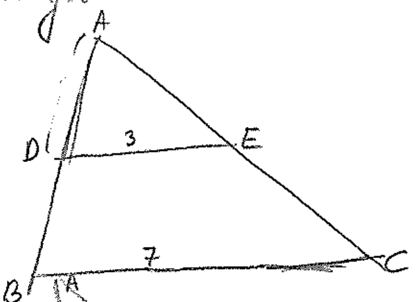
Can we still prove $\frac{Ax+By}{Cx+Dy} = \frac{Ax'+By'}{Cx'+Dy'}$ * (or simpler version of your choice!) ?

Hint In both equation, multiply by some numbers to get rid of the denominators:

$$\frac{x}{y} \cdot yy' = \frac{x'}{y'} \cdot yy' \iff xy' = x'y.$$

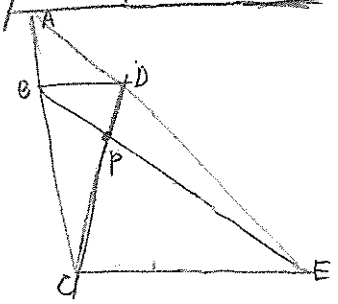
... same with * ...

Applications Example: (... some intro lesson on similar triangles either inserted here or in a previous lesson...)



Find $\frac{AD}{BD}$

or



Other examples?

$\frac{AB}{BC} = \frac{4}{5}$. What's $\frac{BP}{PE}$ = ?

stuff like this maybe set up as an interesting word problem? though it may not be necessary)...

Further on : Products and divisions

4) Division $\rightsquigarrow$ towers of fractions?

What is $\dfrac{1}{\frac{b}{c}} = ?$ (Possibly with b, c = some chosen numbers).

<u>Hint</u>: Find a problem that has $\dfrac{1}{\frac{b}{c}}$ as solution:

$$\boxed{\begin{array}{l}\text{By which number should we multiply } \frac{b}{c} \text{ to get } 1? \\ \boxed{?} \cdot \frac{b}{c} = 1\end{array}}$$

Note that $\dfrac{c}{b} \cdot \dfrac{b}{c} = \dfrac{bc}{bc} = 1.$

So $\boxed{\dfrac{c}{b} = \dfrac{1}{\frac{b}{c}}}$

Follow up: $\dfrac{a}{\frac{b}{c}} = \dots\dots$

$$\frac{\frac{a}{d}}{\frac{b}{c}} = \dots\dots$$

In words: Dividing by a fraction = multiplying by the <u>inverse</u> (? suitable word?) fraction....

<u>Examples</u>: Some "$\boxed{\dfrac{\frac{\square}{\square} + \frac{\square}{\square}}{\frac{\square}{\square} - \frac{\square}{\square}}}$" exercises,

$$\frac{\square}{\frac{\square}{\frac{\square}{\frac{\square}{\frac{\square}{\square}}}}}, \dots$$