Multiple recurrence for dynamical systems on C^*-algebras

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Let $\mathfrak{A}$ be a C^*-algebra, φ a state on $\mathfrak{A}$, and Φ a $*$-endomorphism of $\mathfrak{A}$, which leaves φ invariant. Then the following recurrence property, which corresponds to the classical "Poincaré recurrence" for measure preserving transformations of probability measure spaces, always holds :

for every $0 \leqslant a \in \mathfrak{A}$ *with* $\varphi(a) > 0$ *we have*

$$\varphi\Big(a \cdot \Phi^n(a)\Big) \neq 0 \textit{ for some integer } n \geqslant 1.$$

Much less is known about multiple recurrence in this general setting, that is about the validity, for a given integer $k \geqslant 2$, of the implication

$$0 \leqslant a \in \mathfrak{A}\,,\ \varphi(a) > 0 \implies$$
$$\varphi\Big(a \cdot \Phi^n(a) \cdot \Phi^{2n}(a) \cdots \Phi^{kn}(a)\Big) \neq 0 \textit{ for some integer } n \geqslant 1.$$

In the case of commutative $\mathfrak{A}$, the validity of the above implication for every $k \geqslant 2$ is a deep multiple recurrence result of H. Furstenberg. In this talk we plan to present some results obtained in the case of general $\mathfrak{A}$.