Biological signals: listening to people without cutting them open

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Biological signals

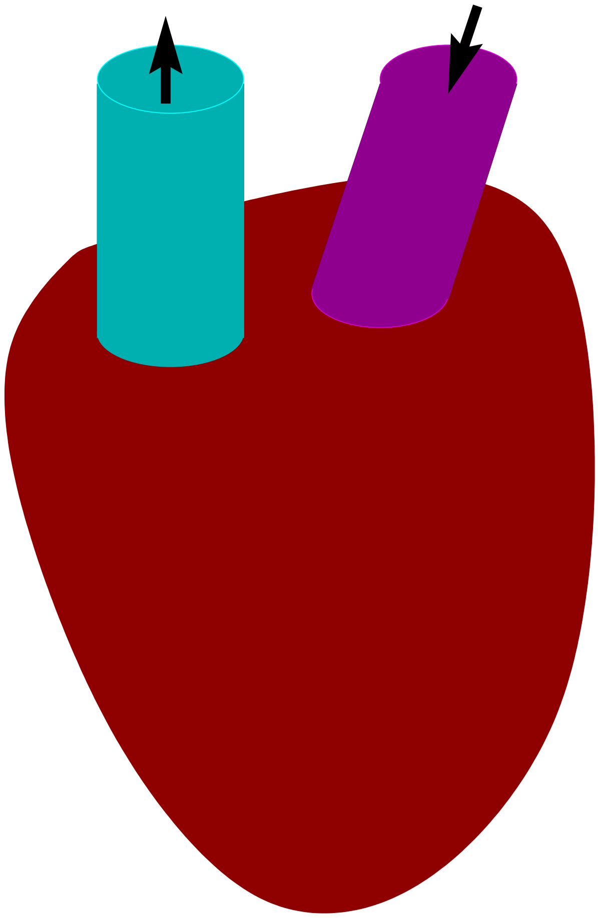
I have worked on

- muscle sounds
- breath sounds
- electrocardiograms (heart electricity)
- ballistocardiograms (heart motion)

All can be measured without cutting people open.

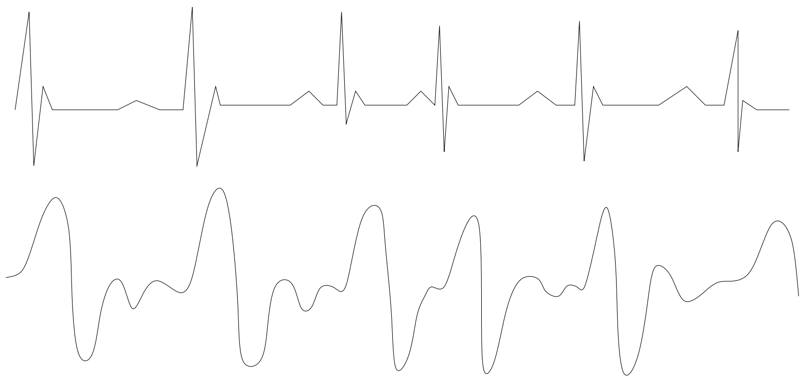
Today's topic: ballistocardiograms (BCGs)

BCG

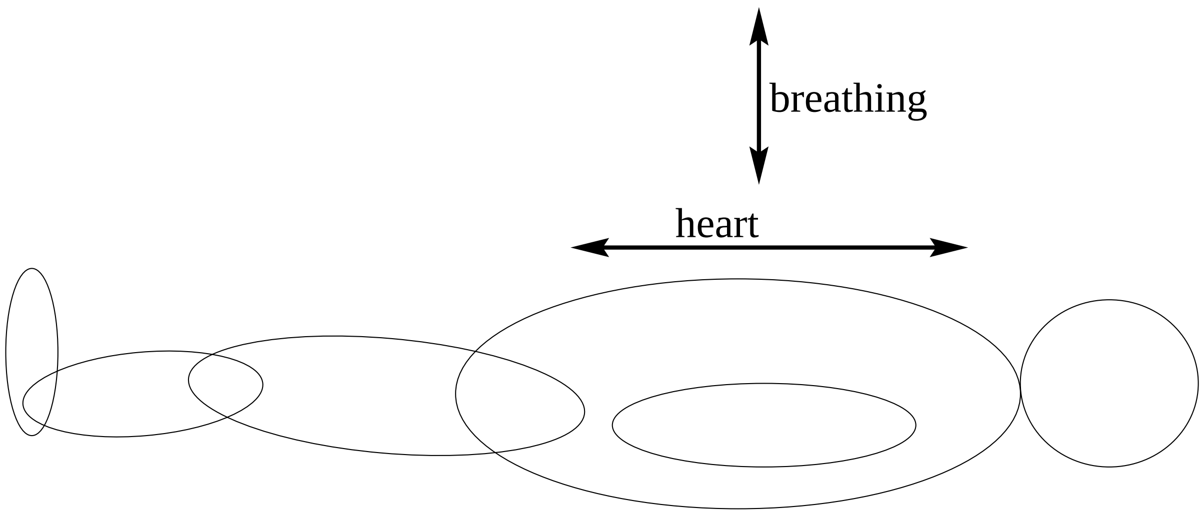


The heart beats, pushing blood up, and pulling it in.

What BCGs look like



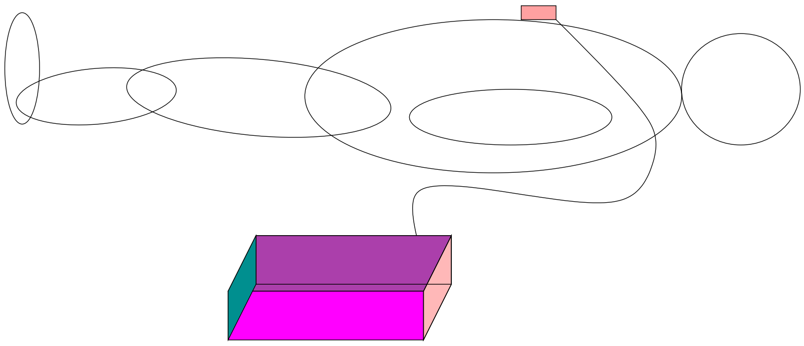
(a) The ECG (electrical trigger from the brain) and
(b) the BCG that results (body acceleration).



How to measure BCGs

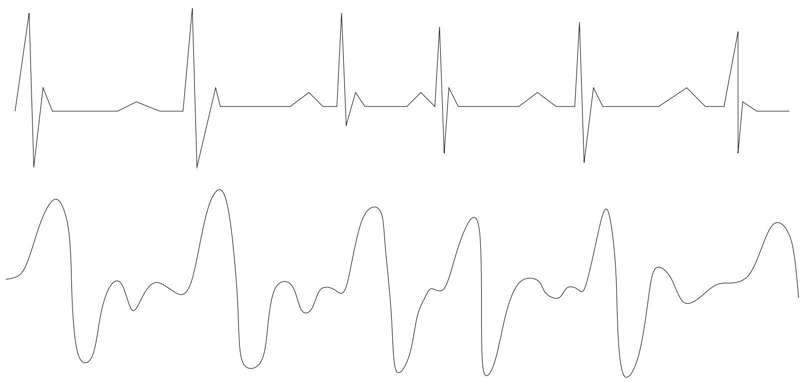
- 18$^{\text{th}}$ century, France, beds on springs, recording by eye
- 1916, France (U.S. Army), beds on springs, recording by light beam & photographic plate
- 1930's, Philadelphia, patient floating on mercury pool, recording by light beam & photographic plate
- 1964, USSR, accelerometer attached to chest, recording by tape
- 1972, USSR, accelerometer attached to cosmonaut in space station, recording by AD converter & computer

Our equipment



Accelerometer—earthquake measuring device, matchbox size (with battery). Feeds into standard VCR, or computer serial port. Fit to patient with double-sided tape.

Signal analysis



Heart beats are not periodic. Forget Fourier.

1. Pick out heart beats from ECG (electric signal from brain that sets off heart beats).
2. Keep BCG signal for only 1/4 second after beat (before next beat).

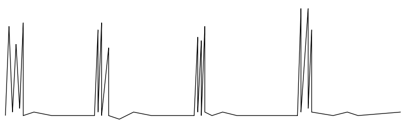
How to find heartbeats in ECG

Before my work, heart beat detectors missed about 1 in 10 beats.

Big problem: slow drift—low frequency

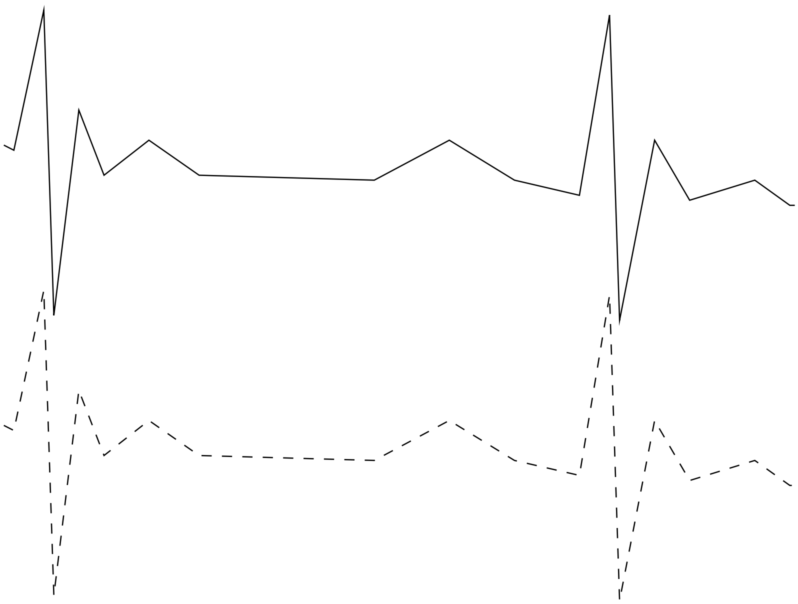
$$\frac{d}{dt}\text{low frequency} = \text{small}$$

Thus $\left(\frac{dECG}{dt}\right)^2$ looks like



Just count high peaks (ignore if too close together). Misses less than 1 in 200 beats.

How to take the derivative of discrete data



$$
\begin{aligned}
\frac{d}{dt} ECG &= \int \delta(t) \frac{d}{dt} ECG \, dt \\
&= - \int ECG(t) \frac{d\delta}{dt} \, dt \\
&\cong - \int ECG(t) \frac{d}{dt} \text{Gaussian}(t) \, dt
\end{aligned}
$$

Works pretty well, with sharp Gaussian, even for digitized signals.

Breaking down into modes

Heart motions are complicated—no good model. How do you study signals without a good model of what generated them?

Wavelet people say: look at the symmetry group of the model, its rep theory, wave a magic wand and out comes the natural way to write system motions as sums of waves. Example: the simple linear wave equation on a circular ring gives Fourier series.

No dice here: symmetry group of every model is $\{1\}$.

Linear algebra I

signal = function = element of Hilbert space

Digitized signals $x(t)$ are just finite lists of numbers:

$$x = \begin{pmatrix} x(0) \\ x(\Delta t) \\ x(2\Delta t) \\ \vdots \end{pmatrix}$$

i.e. vectors, element of finite dim'l Hilbert space

Linear algebra II

Thinking of lots of patients' signals as vectors $x_1, x_2, \ldots$. Find a unit vector u_1 maximizing

$$\sum_p \langle x_p, u_1 \rangle^2 .$$

(Largest correlation to data). So if data is nearly in a straight line, u_1 is the direction of that line.

Linear algebra III

Now find u_2 unit length vector perp to u_1 maximizing

$$\sum_p \langle x_p, u_2 \rangle^2$$

(same thing). Keep going to get $u_1, u_2, u_3, \ldots$.

Linear algebra IV

Not hard to show (Rayleigh–Ritz): if

$$X = [x_1 \quad x_2 \quad \ldots]$$

matrix (with x_j columns) then u_j are eigenvectors of XX^t, with eigenvalues

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq 0.$$

Back to signals

So we get a basis u_j for our Hilbert space, and data lies close to line $\langle u_1 \rangle$, closer to plane $\langle u_1, u_2 \rangle$, etc.

Each signal $x(t)$ is $x(t) = \sum_j a_j u_j(t)$, some coefficients a_j, which are like Fourier coefficients.

Can measure how many coefficients are really needed:

$$\lambda_j = \sum_p \langle u_j, x_p \rangle^2$$

saying how well u_j is correlated to the original x_p signals. So if λ_j is small, then forget about u_j: the data doesn't go very far in that direction.

Back to BCG

For BCG,

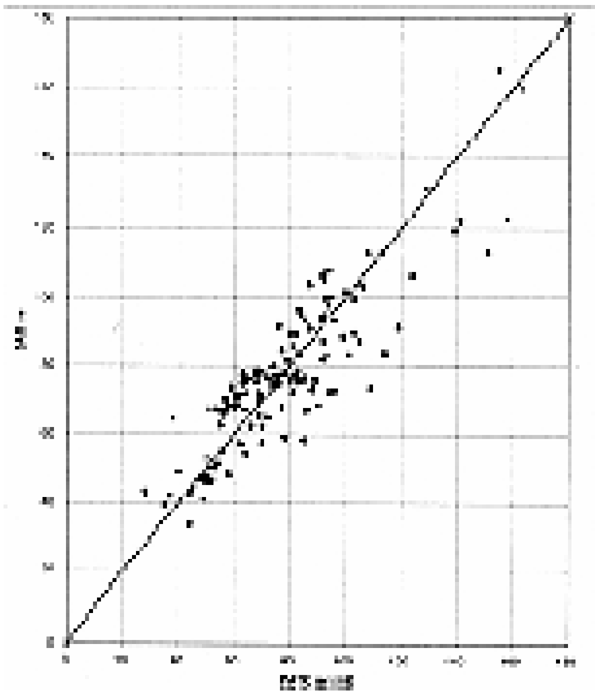
$$\frac{\sum_{j=1}^{7} \lambda_j}{\sum_j \lambda_j} \geq 95\%$$

so we only need the first 7 $u_j(t)$ functions to accurately represent the BCG signal. Each signal has about 2000 data points, so a 7 dimensional vector subspace of a 2000 dimensional space. Reveals the true degrees of freedom in the signals.

Application I: measuring blood flow

We directly measured blood flow (using a tube stuck into the neck going down into the heart): using least squares regression, found a universal linear combination of the 7 coefficients which predicts blood flow for anyone from the 7 a_j numbers:

$$\text{blood flow} = C_0 + \sum_j C_j a_j$$



Roughly as accurate as the tube in the neck, which is not very accurate (experiments in cadavers).

Application II: coronary artery disease

Using another linear combo

$$D_0 + \sum_j D_j a_j > 1$$

says you have coronary artery disease (85% accurate); if < 0 then you don't (82% accurate). Test takes 10 seconds (with signal processing time).

What we learned

- We have a method, using linear algebra, to write signals in an expansion, using only a small number of coefficients.
- Its reveals the true number of degrees of freedom in the original process.
- We can estimate blood flow through the heart from these coefficients.
- We can also detect coronary artery disease.
- Take any model of heart motion, and look at u_j functions it generates, and compare them to our u_j functions we got from the data. Our u_j are characteristics of the physical phenomenon generating the BCG. So far, no model generates reasonable u_j functions.