

# Holomorphic parabolic geometries and Calabi-Yau manifolds

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November 20, 2008

## Abstract

We prove that the only complex parabolic geometries on Calabi-Yau manifolds are the homogeneous geometries on complex tori. We also classify the complex parabolic geometries on homogeneous compact Kähler manifolds.

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## 1 Calabi-Yau manifolds

We will prove that Calabi-Yau manifolds (other than complex tori) cannot bear parabolic geometries. Our arguments are simpler than those of Gunning [5] or Kobayashi [9], and give stronger conclusions (not requiring normalcy, and applying directly to all parabolic geometries, not just projective and conformal connections).

This material is based upon works supported by the Science Foundation Ireland under Grant No. MATF634.

*Definition 1.* For this article, a *Calabi-Yau manifold* is a compact Kähler manifold  $M$  with  $c_1(TM) = 0$ .

**Lemma 1.** *A Calabi-Yau manifold satisfies  $c_2(TM) = 0$  just if it has a torus as unramified covering space.*

*Proof.* For any Kähler manifold, say of dimension  $n$ , with  $\Omega$  its Kähler form, it is easy to calculate that

$$c_2 \wedge \Omega^{n-2} = \left( \|R\|^2 + \text{scalar}^2 - 2\|\text{Ricci}\|^2 \right) \Omega^n,$$

(see Lascoux and Berger [10]) where  $R$  is the curvature tensor. If  $c_1 = 0$ , then there is a metric for which  $\text{Ricci} = 0$ , by Yau's solution of the Calabi conjecture [16]. Hence  $c_2 = 0$  implies  $R = 0$ , flat. But then  $M$  is covered by a flat torus (see Igusa [6]).  $\square$

**Lemma 2** (Inoue, Kobayashi and Ochiai [7]). *Any compact complex manifold which bears a holomorphic Cartan geometry with reductive algebraic structure group has vanishing Atiyah class. In particular, if Kähler then it is the quotient of a complex torus under a finite unramified covering map.*

*Proof.* The Cartan connection splits invariantly into a sum of a connection (in the sense of Ehresmann) and a soldering form; see Sharpe [15] p.362 lemma 2.1. The existence of a connection is precisely the vanishing of the Atiyah class; see Atiyah [1]. If Kähler, then all Chern classes of the tangent bundle vanish just when the Atiyah class does. By the previous lemma, the manifold has a torus as finite unramified covering space.  $\square$

*Example 1.* A holomorphic Riemannian metric is a simple example of a reductive Cartan geometry, and our results tell us that holomorphic Riemannian metrics can not live on any compact Kähler manifold except those covered by tori. This is well known (see Inoue, Kobayashi and Ochiai [7]).

**Theorem 1.** *If a Calabi–Yau manifold bears a parabolic geometry, then it is covered by a torus. More generally, any compact complex manifold with a parabolic geometry and trivial canonical bundle must have vanishing Atiyah class.*

Replacing our Calabi-Yau manifold by a finite covering space if needed, we can assume that it bears a nowhere-vanishing holomorphic volume form. We then derive our theorem from the following stronger theorem:

**Theorem 2.** *If a complex manifold bears a holomorphic parabolic geometry and a holomorphic volume form, then it admits a canonical holomorphic reduction of the structure group of the parabolic geometry to a reductive algebraic group.*

*Proof.* Suppose that  $E \rightarrow M$  is a parabolic geometry modelled on  $G/P$ , and  $\sigma$  a holomorphic volume form on  $M$ . Recall that  $G/P$  is a rational homogeneous variety, so  $G$  is a complex semisimple Lie group and  $P$  is a parabolic subgroup. We can express the Lie algebra of  $P$  as a sum of the Cartan subalgebra of  $G$  together with various root spaces, including all of the positive root spaces. Some negative root spaces may also lie in the Lie algebra of  $P$ . Roots whose root space does not in the Lie algebra of  $P$  are called *omitted roots* of  $P$ . The roots divide up into 3 types: (1) omitted roots, (2) roots which are not omitted but for which  $-\alpha$  is omitted, called nilpotent roots, and (3) roots for which neither  $\alpha$  nor  $-\alpha$  are omitted, called reductive roots. The maximal reductive (nilpotent) subalgebra  $\mathfrak{m} \oplus \mathfrak{a}(\mathfrak{n})$  of  $P$  is the sum of reductive (nilpotent) root spaces; see Knapp [8]. The splitting  $\mathfrak{g} = \mathfrak{n}^- \oplus \mathfrak{m} \oplus \mathfrak{a}/\mathfrak{n}$  is  $MA$ -invariant, with  $MA$  the corresponding maximal reductive subgroup of  $P$ . The groups  $M$ ,  $A$ ,  $N$  and  $P$  are all algebraic (see Fulton and Harris [4] p. 382).

Pick a Chevalley basis  $X_\alpha, H_\alpha$  for  $\mathfrak{g}$  (the Lie algebra of  $G$ ). Recall (see Serre [14]) that this is a basis parameterized by roots  $\alpha \in \mathfrak{h}^*$  (with  $\mathfrak{h} \subset \mathfrak{g}$  a Cartan subalgebra) for which

1.  $[H, X_\alpha] = \alpha(H)X_\alpha$  for each  $H \in \mathfrak{h}$
2.  $\alpha(H_\beta) = 2 \frac{\langle \alpha, \beta \rangle}{\langle \beta, \beta \rangle}$  (measuring inner products via the Killing form)
3.  $[H_\alpha, H_\beta] = 0$ ,

4.

$$[X_\alpha, X_\beta] = \begin{cases} H_\alpha, & \text{if } \alpha + \beta = 0, \\ N_{\alpha\beta}X_{\alpha+\beta}, & \text{otherwise} \end{cases}$$

with

- (a)  $N_{\alpha\beta}$  an integer,
- (b)  $N_{-\alpha, -\beta} = -N_{\alpha\beta}$ ,
- (c) If  $\alpha, \beta$ , and  $\alpha + \beta$  are roots, then  $N_{\alpha\beta} = \pm(p + 1)$ , where  $p$  is the largest integer for which  $\beta - p\alpha$  is a root,
- (d)  $N_{\alpha\beta} = 0$  if  $\alpha + \beta = 0$  or if any of  $\alpha, \beta$ , or  $\alpha + \beta$  is not a root.

Consider the 1-forms  $\omega^\alpha$  dual to the vectors  $X_\alpha$  of a Chevalley basis. We use the Killing form to extend  $\alpha$  from  $\mathfrak{h}$  to  $\mathfrak{g}$ , by splitting  $\mathfrak{g} = \mathfrak{h} + \mathfrak{h}^\perp$ , and taking  $\alpha = 0$  on  $\mathfrak{h}^\perp$ . The 1-forms  $\omega^\alpha, \alpha$  span  $\mathfrak{g}^*$ .

Each exterior form in  $\Lambda^*(\mathfrak{g})^*$  extends uniquely to a left invariant differential form in  $\Omega^*(G)$ , and we will identify these. These forms determine a basis of left invariant 1-forms  $\omega^\alpha, \alpha$ , and a basis of left invariant vector fields  $X_\alpha, H_\alpha$ . Clearly

$$d\omega^\alpha = -\alpha \wedge \omega^\alpha - \frac{1}{2} \sum_{\beta+\gamma=\alpha} N_{\beta\gamma} \omega^\beta \wedge \omega^\gamma$$

$$d\alpha = -\sum_{\beta} \frac{\langle \alpha, \beta \rangle}{\langle \beta, \beta \rangle} \omega^\beta \wedge \omega^{-\beta},$$

with sums over all roots. To be more precise  $\omega^\alpha, \alpha$  is not quite a basis of 1-forms, since there will be relations among the  $\alpha$  1-forms in general. To produce a basis, we would have to restrict to the  $\alpha$  1-forms which are simple roots, but include all of the  $\omega^\alpha$  1-forms, even for nonsimple  $\alpha$ . The basis  $\omega^\alpha, \alpha$  is *not* the dual basis to  $X_\alpha, H_\alpha$ .

Very similar structure equations hold for any parabolic geometry with the same model. Indeed, the Cartan connection is a 1-form valued in the Lie algebra  $\mathfrak{g}$  of  $G$ , so splits into a sum of 1-forms  $\omega^\alpha$  and  $\alpha$  from the decomposition of  $\mathfrak{g}$  into root spaces. From the definition of a Cartan geometry, the Cartan connection satisfies the same structure equations as the Maurer–Cartan form on the model, but with semibasic curvature correction terms, so

$$d\omega^\alpha = -\alpha \wedge \omega^\alpha - \frac{1}{2} \sum_{\beta+\gamma=\alpha} N_{\beta\gamma} \omega^\beta \wedge \omega^\gamma + \sum_{\beta, \gamma} \kappa_{\beta\gamma}^\alpha \omega^\beta \wedge \omega^\gamma,$$

$$d\alpha = -\sum_{\beta} \frac{\langle \alpha, \beta \rangle}{\langle \beta, \beta \rangle} \omega^\beta \wedge \omega^{-\beta} + \sum_{\beta, \gamma} \lambda_{\beta\gamma}^\alpha \omega^\beta \wedge \omega^\gamma,$$

where the  $\kappa$  and  $\lambda$  terms are Cartan geometry curvature terms, so they vanish except possibly for  $\beta$  and  $\gamma$  omitted roots. It is vital in the following that, even if we work on a manifold where we have imposed some relations on these 1-forms, we will still use the Killing form on the original Lie algebra  $\mathfrak{g}$  to compute inner products  $\langle \alpha, \beta \rangle$ . This is our only notational ambiguity.

Let

$$\begin{aligned}\Omega &= \bigwedge_{\alpha} \omega^{\alpha}, \\ \delta &= \frac{1}{2} \sum_{\alpha} \alpha,\end{aligned}$$

where the wedge product and sum are over omitted roots. The sign of  $\Omega$  depends on a choice of ordering of the omitted roots, but any ordering can be chosen, as long as we are consistent. Our nonzero section  $\sigma$  of the canonical bundle of  $M$  can be pulled back to  $E$  as  $\sigma = s\Omega$ , for a unique nowhere-vanishing function  $s : E \rightarrow \mathbb{C}$ . If  $\sigma$  is holomorphic, then

$$\begin{aligned}0 &= d\sigma \\ &= ds \wedge \Omega + s d\Omega\end{aligned}$$

Ordering the omitted roots as  $\alpha_1, \alpha_2, \dots$ :

$$= ds \wedge \Omega + s \sum_j (-1)^{j+1} \bigwedge_{i < j} \omega^{\alpha_i} \wedge d\omega^{\alpha_j} \wedge \bigwedge_{i > j} \omega^{\alpha_i}$$

If  $\beta + \gamma = \alpha$  and  $\alpha$  is omitted, then one of  $\beta$  or  $\gamma$  must be as well; say  $\beta$ . But  $\gamma \neq 0$  since  $\gamma$  is also a root. Therefore  $\beta$  is an omitted root other than  $\alpha_j$ . So these terms inside  $d\omega^{\alpha_i}$  die off. The other sum in  $d\omega^{\alpha_i}$  also dies off, because the terms must involve wedge products of distinct omitted roots, so each term has an omitted root other than  $\alpha_j$ :

$$= (ds - 2s\delta) \wedge \Omega.$$

Let  $E' \subset E$  be the set of points at which  $s = 1$ , a smooth hypersurface since  $ds \neq 0$  on tangent spaces of  $E$  along  $E'$ . In fact,  $E'$  is a principal right  $P_0$ -bundle, where  $P_0$  is the subgroup of the structure group  $P$  preserving a volume form on  $\mathfrak{g}/\mathfrak{p}$ . On  $E'$ ,  $\delta \wedge \Omega = 0$ . Therefore  $\delta$  is semibasic on  $E'$ :

$$\delta = \sum_{\alpha} t_{\alpha} \omega^{\alpha},$$

a sum over omitted roots  $\alpha$ , for some functions  $t_{\alpha} : E' \rightarrow \mathbb{C}$ . Taking exterior derivative, we find

$$\begin{aligned}0 &= d \left( \delta - \sum_{\alpha} t_{\alpha} \omega^{\alpha} \right) \\ &= \sum_{\alpha} (d\alpha - dt_{\alpha} \wedge \omega^{\alpha} - t_{\alpha} d\omega^{\alpha}) \\ &= - \sum_{\beta} \frac{\langle \delta, \beta \rangle}{\langle \beta, \beta \rangle} \omega^{\beta} \wedge \omega^{-\beta} - \sum_{\alpha} (dt_{\alpha} - t_{\alpha} \alpha) \wedge \omega^{\alpha} \\ &\quad - \frac{1}{2} \sum_{\alpha} t_{\alpha} \sum_{\beta + \gamma = \alpha} N_{\beta\gamma} \omega^{\beta} \wedge \omega^{\gamma} \quad (\text{mod semibasic terms})\end{aligned}$$

In particular, for any omitted root  $\alpha$ ,

$$\mathcal{L}_{X_{-\alpha}} t_\alpha = 2 \frac{\langle \delta, \alpha \rangle}{\langle \alpha, \alpha \rangle}.$$

The vector field  $X_{-\alpha}$  is tangent to the fibers of  $E' \rightarrow M$ . On the fibers the vector field  $X_{-\alpha}$  is a left invariant vector field. Therefore  $X_{-\alpha}$  is complete. Starting at any point of  $E'$ , we can move in the direction  $X_{-\alpha}$  of the nilpotent part of the structure group, altering the value of  $t_\alpha$  at a constant rate until it reaches 0. Indeed  $t_\alpha$  is acted on by the nilradical of the structure group as translations in the left action on  $E'$ . The set of points  $E'' \subset E'$  on which all  $t_\alpha$  vanish is a smooth embedded submanifold, because its tangent space is cut out by equations

$$\frac{\langle \delta, \alpha \rangle}{\langle \alpha, \alpha \rangle} \omega^{-\alpha} = \text{semibasic},$$

for all omitted roots  $\alpha$ . The structure group is reduced to a reductive algebraic group, since we have eliminated the nilradical of the original structure group, leaving only the root spaces  $\alpha$  for which neither  $\alpha$  nor  $-\alpha$  is omitted, i.e. the root spaces of the maximal reductive subgroup  $MA$  of the structure group  $P$ . We have also eliminated the part of  $A$  which acts nontrivially on the holomorphic volume forms, so our structure group is now  $MA^0$ , with  $A^0$  the subgroup of  $A$  fixing a volume form on  $\mathfrak{g}/\mathfrak{p}$ .  $\square$

*Remark 1.* On a complex manifold with a meromorphic section of the canonical bundle, it would be interesting to consider what happens to this argument as we approach the zeroes or poles of the meromorphic section.

*Remark 2.* Previously I proved [11] that Calabi–Yau manifolds which contain rational curves cannot bear a holomorphic Cartan geometry. Above I have removed the need for rational curves, but at a price of only being able to exclude parabolic geometries. It seems a natural conjecture that Calabi–Yau manifolds bear no holomorphic Cartan geometry. Dumitrescu [3] has similar results for affine geometries.

## 2 Parabolic geometries on tori

*Example 2.* Pick  $P \subset G$  a closed subgroup of a Lie group, with Lie algebras  $\mathfrak{p} \subset \mathfrak{g}$ . Take any linear subspace  $\Pi \subset \mathfrak{g}$  transverse to  $\mathfrak{p}$ . Take  $\Gamma : \mathfrak{g}/\mathfrak{p} \rightarrow \mathfrak{g}$  the linear section of the obvious linear map  $\mathfrak{g} \rightarrow \mathfrak{g}/\mathfrak{p}$  with image  $\Pi$ . Let  $M = \mathfrak{g}/\mathfrak{p}$ ,  $E = M \times P$ . Writing elements of  $E$  as  $(x, h) \in E$ , let

$$\omega = h^{-1} dh + \text{Ad}_h^{-1} (\Gamma(dx)),$$

a translation invariant Cartan geometry on  $\mathfrak{g}/\mathfrak{p}$ . Every translation invariant Cartan geometry on a vector space is isomorphic to this one for some choice of  $\Pi$ . This Cartan geometry is flat just when  $\Pi \subset \mathfrak{g}$  is an abelian subgroup. In particular, unless  $\mathfrak{g}$  is abelian or  $\mathfrak{p} = \mathfrak{g}$ , we can always find a subspace  $\Pi \subset \mathfrak{g}$  for which the induced Cartan geometry is *not* flat. By translation invariance, this Cartan geometry induces a curved Cartan geometry on any torus or torus quotient of the required dimension.

For example, if  $\mathfrak{g}$  is semisimple then we can take  $\Pi$  to be the orthogonal complement to  $\mathfrak{p} \subset \mathfrak{g}$ . Alternatively, if  $\mathfrak{p}$  is reductive, we can take  $\Pi$  to be a complementary  $\mathfrak{p}$ -invariant subspace to  $\mathfrak{p} \subset \mathfrak{g}$ .

**Theorem 3.** *Every parabolic geometry on any complex torus is translation invariant, and obtained by the construction of example 2 on the preceding page.*

*Proof.* Let  $M$  be a torus and  $E \rightarrow M$  a parabolic geometry. After the calculations of theorem 1 on page 2, the structure group of the parabolic geometry reduces to a reductive group,  $G''$  on some subbundle  $E'' \subset E$ . The Cartan connection  $\omega$  splits into a sum corresponding to the splitting of  $\mathfrak{g}$  into  $G''$ -invariant subspaces, and  $\omega''$  (the part valued in  $\mathfrak{g}''$ ) is a connection form for  $E'' \rightarrow M$ . Take a global coframing on  $M$ , i.e. a set of linearly independent 1-forms  $\xi^\alpha$  forming a basis of each cotangent space of  $M$ , for  $\alpha$  varying over omitted roots. Define a map  $e \in E'' \rightarrow h \in \mathrm{GL}(\mathfrak{g}/\mathfrak{p})$ , by  $\omega^\alpha = h_\beta^\alpha \xi^\beta$  (for  $\alpha$  and  $\beta$  varying over omitted roots), and  $h(e) = \left( h_\beta^\alpha \right)$  in the basis  $X_\alpha$  for the sum of omitted root spaces. Under right  $G''$ -action,

$$h(r_g e) = g^{-1} h(e)$$

for  $g \in G''$ . Therefore the quotient map  $E \rightarrow \mathrm{GL}(\mathfrak{g}/\mathfrak{p})/G''$  descends to a map  $M \rightarrow \mathrm{GL}(\mathfrak{g}/\mathfrak{p})/G''$ . The quotient  $\mathrm{GL}(\mathfrak{g}/\mathfrak{p})/G''$  is an affine variety: see Mumford et. al. [12] p.27 theorem 1.1 and Procesi [13] p. 556, theorem 2. Affine coordinate functions will pull back to functions on the torus  $M$ , and therefore must be constant. Therefore the map  $M \rightarrow \mathrm{GL}(\mathfrak{g}/\mathfrak{p})/G''$  is constant. Since the coframing  $\xi^\alpha$  is arbitrary, we can arrange that  $h(e) = 1$  at some point of  $E''$ , identifying  $T_m M$  with  $\mathfrak{g}/\mathfrak{p}$ . We have an isomorphism

$$e \in E'' \rightarrow (\pi(e), h(e)) \in M \times G'',$$

trivializing the bundle  $E''$ . We identify  $M$  with  $(\mathfrak{g}/\mathfrak{p})/\Lambda$  for some lattice  $\Lambda \subset \mathfrak{g}/\mathfrak{p}$ . Then we let  $\Pi$  be the image of  $\omega$  at  $(0, 1) \in M \times G''$ .  $\square$

**Corollary 1.** *If a compact Kähler manifold with  $c_1 = 0$  bears a parabolic geometry, then it is covered by a torus, and the parabolic geometry pulls back to a translation invariant parabolic geometry on the torus.*

*Definition 2.* Suppose that  $P_- \subset P_+$  are two closed subgroups of a Lie group  $G$ , so that we have a fiber bundle map  $G/P_- \rightarrow G/P_+$ . Let  $W$  be a  $P_+$ -module, and  $E \rightarrow M$  a Cartan geometry modelled on  $G/P_+$ . Then  $E \rightarrow E/P_-$  is a Cartan geometry modelled on  $G/P_-$ , called the *lift* of the Cartan geometry on  $M$ .

**Corollary 2.** *On any compact homogeneous Kähler manifold, all parabolic geometries are lifted (as in definition 2) from a translation invariant geometry on a torus (constructed as in example 2 on the previous page). In particular, all such parabolic geometries are homogeneous.*

*Proof.* Borel and Remmert [2] proved that every compact homogeneous Kähler manifold is a product of a torus and a rational homogeneous variety. The rational homogeneous variety bears rational curves just when it has positive dimension. These rational curves ensure that the parabolic geometry is lifted from lower dimension (see McKay [11]), quotienting out the rational homogeneous variety entirely.  $\square$

*Remark 3.* Any parabolic geometry on any rational homogeneous variety is flat and isomorphic to its model (see McKay [11]).

### 3 Conclusion

*Conjecture 1.* If  $M$  is a compact complex manifold with  $c_1 < 0$ , then either (1)  $M$  admits no parabolic geometry, or (2)  $M$  admits a parabolic geometry modelled on a compact Hermitian symmetric space  $G/P$  and  $M$  is covered by the noncompact dual of that symmetric space. In case (2), every parabolic geometry on  $M$  modelled on  $G/P$  is flat.

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