

Coláiste An Spioraid Naomh Maths Circle
Lesson 9

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Warm Up Calculation

- Step1: Add 18 to your birth month.
- Step2: Multiply by 25.
- Step3: Subtract 333.
- Step4: Multiply by 8.
- Step5: Subtract 554.
- Step6: Divide by 2.
- Step7: Add your birth date.
- Step8: Multiply by 5.
- Step9: Add 692.
- Step10: Multiply by 20.
- Step11: Add only the last two digits of your birth year.
- Step12: Subtract 32940 to get your birthday!.

Example: If the answer is 123199 means that you were born on December 31, 1999.

1. Sicherman Dice



These dice at first glance seem very random. The faces on the dice are numbered 1,2,2,3,3,4 and 1,3,4,5,6,8. Roll the dice a few times and see can you find anything interesting about them.

Hint: Make out a table to see what numbers you can score from rolling the two dice and compare this to that of a standard pair of dice.

Solution:

	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

	2	3	3	4	4	5
	4	5	5	6	6	7
	5	6	6	7	7	8
	6	7	7	8	8	9
	7	8	8	9	9	10
	9	10	10	11	11	12

Sicherman dice are the only pair of 6-sided dice that are not normal dice, bear only positive integers, and have the same probability distribution for the sum as normal dice. So, if you want to play a board game that requires a pair of dice, but don't have any, but (for some reason) have a pair of Sicherman dice, you can still play!!

These dice were discovered by George Sicherman and were originally reported by Martin Gardner in a 1978 article in Scientific American.

The numbers are arranged so that all pairs of numbers on opposing sides sum to equal numbers, 5 for the first and 9 for the second.

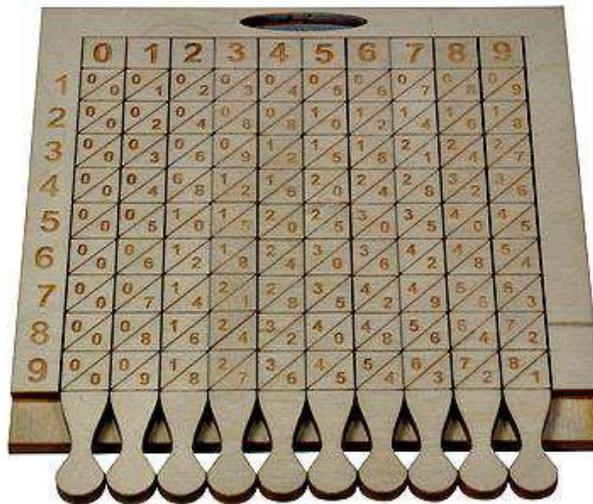
Can you think of any other sets of numbers (including negative numbers) that have the same probability distribution?

Solution:

This is easy- just take either a pair of regular dice or a pair of Sicherman dice, and add any number to each of the numbers on one of the dice, and subtract the same number from each of the numbers on the other.

2. Napier Bones

John Napier was a Scottish Mathematician, most famous for discovering Logarithms. He also invented a smart tool for doing long multiplication and division in 1617, called Napier Bones.



The abacus consists of a board with a rim; the user places Napier's rods in the rim to conduct multiplication or division. The board's left edge is divided into 9 squares, holding the numbers 1 to 9. Each of the rods are divided in nine squares, with each square having the corresponding multiple of that rod's base number. However the digits of each product are separated by a diagonal line.

The best way to learn how to use Napier bones is by seeing an example: Suppose, you want to multiply 425,928 by 7. We place the 4, 2, 5, 9, 2 and 8 rods on the board. We then look at the 7th row.

	4	2	5	9	2	8
2	0/8	0/4	1/0	1/8	0/4	1/6
3	1/2	0/6	1/5	2/7	0/6	2/4
4	1/6	0/8	2/0	3/6	0/8	3/2
5	2/0	1/0	2/5	4/5	1/0	4/0
6	2/4	1/2	3/0	5/4	1/2	4/8
7	2/8	1/4	3/5	6/3	1/4	5/6
8	3/2	1/6	4/0	7/2	1/6	6/4
9	3/6	1/8	4/5	8/1	1/8	7/2

We move from right to left, adding the numbers between each set of diagonal lines. For “5 + 6”, we write “1” and carry 1 to the next column. We get 2,981,496.

Use the Napier Bones to do the following multiplications and check your answers using your calculators:

- $5 \times 275,931$
- $4 \times 712,989$
- $8 \times 187,992$

We can also multiply 2 numbers both with multiple digits. We copy down the appropriate rows from the Napier bones, and then add along the diagonals similar to before. Again, it’s best understood from seeing an example- Suppose we are multiplying 425,928 by 56:

	4	2	5	9	2	8
2	0 8	0 4	1 0	1 8	0 4	1 6
3	1 2	0 6	1 5	2 7	0 6	2 4
4	1 6	0 8	2 0	3 6	0 8	3 2
5	2 0	1 0	2 5	4 5	1 0	4 0
6	2 4	1 2	3 0	5 4	1 2	4 8
7	2 8	1 4	3 5	6 3	1 4	5 6
8	3 2	1 6	4 0	7 2	1 6	6 4
9	3 6	1 8	4 5	8 1	1 8	7 2

Again, reading from right to left, this time adding the four numbers between each pair of diagonal lines we get 23,851,968.

Use the Napier Bones to do the following multiplications and check your answers using your calculators:

- $45 \times 365,971$
- $78 \times 122,089$
- $234 \times 187,260$

Take Home Problem

3 men checked into a hotel one day- the price was €30, so they each paid €10 and went to their room. The hotel manager then remembered that they had a special offer that night- €25 for a room. So he sent the bellboy up to the room with the €5 that he overcharged them- 5 €1 coins. The three men couldn't split the five coins between them so they each took €1 and gave the bellboy a €2 tip.

Because they each got €1 back, in fact, each of the men paid €9. $3 \times €9 = €27$, adding on the €2 tip we get, $€27 + €2 = €29$. Where did the missing euro go???