Lesson Plan Week 9

Introduction
This is a puzzle station lesson with three puzzles: Skydivers Problem , Cheryl's Birthday Problem and Fun Problems & Paradoxes

Resources
Calculators, pens and paper is all that is needed as well as a copy of the Puzzle station question sheets for each student.

Activities
Warm-up problem(5-10 minutes).
Puzzle Stations(10-15minutes each)

- **SkyDivers Problem** Students learn how to establish patterns and make claims and test the validity of their claim.
- **Cheryl's Birthday Problem** This is a challenging logic puzzle that tests students problem solving abilities
- **Fun Problems & Paradoxes** These problems cause students to think about some important math concepts without having to formally teach any of the concepts. These include Intermediate Value Theorem, Selection Effect in statistics and the concept of infinity.

Warm-up Problem
What digit is the most frequent between the numbers 1 and 1,000 (inclusive)?

*Please note i have included a warm-up problem this can be used as the take-home problem as I presume the warm-up problem will be taken from previous lesson plans take home problem as per usual.

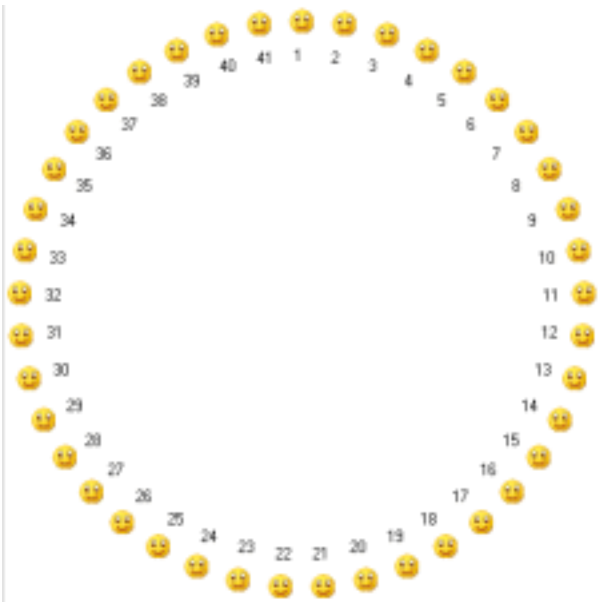
Skydivers Problem

This puzzle is based on a problem named after Flavius Josephus, a Jewish historian living in the 1st century (you may find more information about the origins of this problem by following this link https://en.wikipedia.org/wiki/Josephus_problem).

We're going to put a modern twist on the original problem! A group of friends decide to go skydiving. Before jumping out of the airplane the friends decide to play a game to determine the order in which they jump. They arrange themselves in a circle with positions 1 to n (n being the number of people in the group). Once arranged in this circle the person in position 1 pushes the person to their right out of the plane, then the next person in the circle (ie. position 3) pushes the person to their right out of the plane and so on. This pattern continues until there's only one person left who then jumps out of the airplane themselves.

One of the friends Billy gets nervous and decides he doesn't want to jump. Billy knows that if he's the last person in the game he can stay in the airplane and arrive back to land without his friends knowing he didn't jump. What position should Billy pick in the circle so that he can be the last person?

1. Suppose there's only 2 people in the group what position should Billy pick?
2. Now suppose there's 3,4,5,6,7,8,9…..16 what position should Billy pick in each case?
3. Can you spot a pattern? (hint: which values give you starting position 1?)
4. What position would you pick for a group of 41?



Cheryl's Birthday Problem

You may have heard of this problem before which became popular after being published in a Singapore math contest.

Albert and Bernard just met Cheryl. "When's your birthday?" Albert asked Cheryl.
Cheryl thought a second and said, "I'm not going to tell you, but I'll give you some clues." She wrote down a list of 10 dates:

May 15, May 16, May 19

June 17, June 18

July 14, July 16

August 14, August 15, August 17

"My birthday is one of these," she said.

Then Cheryl whispered in Albert's ear the month — and only the month — of her birthday. To Bernard, she whispered the day, and only the day.
"Can you figure it out now?" she asked Albert.

Albert: I don't know when your birthday is, but I know Bernard doesn't know, either.
Bernard: I didn't know originally, but now I do.
Albert: Well, now I know, too!

When is Cheryl's birthday?

(Hint: Write out all possibilities in a table with months on the left hand side and dates on top. Using statements made by both Albert and Bernard eliminate certain dates and months.)

Fun Problems and Paradoxes

1.Can you explain why at any one time the temperature at two opposite points on Earth have the same temperature? (Assume daytime temperatures are warmer that nighttime temperatures)

2.During World War II some aircraft would arrive back to an airbase after battle and some wouldn't. Of those that arrived back the number of bullet holes for each part of the aircraft were counted and recorded. The data was given to a statistician who was asked to recommend where they should reinforce armour on their planes. The statistician replied ,"The regions with the least amount of bullet holes". Can you explain why the statistician was correct?

3. This problem is known as Zeno's paradox: Achilles and the tortoise. Achilles and a tortoise have a race, the tortoise gets a 10m head start on achilles then achilles starts to run. To pass the tortoise achilles must first cover the 10m between them. Achilles does this but in the mean time the tortoise has moved 5m further. Again achilles makes up the distance but the tortoise has moved another 2.5m. The gap between achilles and the tortoise keeps getting smaller but does achilles ever pass the tortoise?(Assume Achilles is twice as fast as the tortoise)

Solutions

Warm-up Problem: Without manually writing out all the possible numbers we can establish a pattern and see that the numbers 2-9 each occur 300 times while 1 occurs 301 times. We can disregard 0 which only occurs 192 times. Therefore the answer is 1.

Skydiver Problem: Students need to establish a pattern in order to solve this puzzle for larger numbers. The best strategy for doing this is to first make a table of the starting position for a given n and see if we can learn from this data and establish a pattern.

Number of people in group	Starting position in order to be last person left
1	1
2	1
3	3
4	1
5	3
6	5
7	7
8	1
9	3
10	5
11	7
12	9
13	11
14	13
15	15
16	1

The first thing to notice is that a starting position of 1 is achieved when the number of people in the group n is a power of 2. We can claim that if $n= 2^a$ then the winning position w(n) = 1, we will not prove this explicitly however students may accept this fact to be true. We must now figure out what happens with the numbers such that n cannot be represented by 2^a such an n may be represented as $n= 2^a + b$. What we may observe is that after completing b steps in our cycle we will be left with a group size n' which is a power of n. We know that w(n)=starting position for $n= 2^a$ therefore whatever number we land on after b steps is our winning position (ie. whoever turn is next after b steps is in the winning position). This position will always be 2b + 1 since we're moving two positions after each turn and then plus one so we land on the person after the last eliminated as in the persons whose term it is next. Using these facts we may compute the answer for 41 as follows. $41 = 2^5 + 9$, therefore the winning position in 2(9)+1= 19

Cheryl's Birthday Problem

The table below represents the dates that Cheryl listed as possible birthday dates. We will now begin to reduce the possible dates using statements made by both Albert and Bernard.

	14	15	16	17	18	19	Total
May		x	x			x	3
June				x	x		2
July	x		x				2
August	x	x		x			3
Total	2	2	2	2	1	1	

Albert first says,"I don't know when your birthday is, but I know Bernard doesn't know, either." From this statement we can deduce that neither May 19th nor June 18th is possible. This is because Albert says he knows that Bernard doesn't know the birthday, however if the birthday was the on the 18th or 19th Bernard would know the date.

	14	15	16	17	18	19	Total
May		x	x			0	2
June				x	0		1
July	x		x				2
August	x	x		x			3
Total	2	2	2	2	0	0	

Bernard the says," I didn't know originally, but now I do". From Albert's last statement Bernard can immediately deduce that the birthday is neither in the month of May nor June. This comes from the fact that Albert said he knew Bernard didn't know the birthday.

	14	15	16	17	18	19	Total
May		0	0			0	0
June				0	0		0
July	x		x				2
August	x	x		x			3
Total	2	1	1	1	0	0	

If Bernard knows the birthday it must be either August 15th , July 16th or August 17th as these are the only possible dates he can with certainty say occur given the day and Albert's statement.

	14	15	16	17	18	19	Total
May		0	0			0	0
June				0	0		0
July	0		x				1
August	0	x		x			2
Total	2	1	1	1	0	0	

Albert then says," Well, now I know, too!". From this piece of information we can deduce that the date is July 16th as it's the only possible outcome. If it were to be in the month of August we wouldn't be able to determine a month.

	14	15	16	17	18	19	Total
May		0	0			0	0
June				0	0		0
July	0		x				1
August	0	0		0			0
Total	0	0	1	0	0	0	

*We must bare in mind that alternate solutions are possible depending on how we interpret the statements. One such alternate solution is August 17th.

Fun Problems and Paradoxes: 1. This is a useful way to introduce the Intermediate Value Theorem informally. We start by assuming that one side is warmer than the other say the side facing the sun (ie. the side where it's daytime). Choosing a point along the equator facing the sun and another point on the opposite side imagine what happens as the earth begins to rotate. The temperatures should decrease/increase at a constant rate. Lets take the difference between the temperatures at a given time to be x. At the start this difference should be positive but by the time the earth has rotated 180 degrees this difference should be negative (assuming its cooler at night and warmer during the day). Since the rate at which the temperature changes is constant it must pass through 0 at some stage IVT. Therefore we can conclude that the temperature is the same at two opposite points on Earth at a given time. (You may also explain to students the many underlying assumptions made in this problem.)

2. This problem introduces the idea of the selection effect. The airplanes arriving back to the base are the airplanes that survived aerial warfare. Therefore the bullet holes they incurred may be seen as not critical since the aircraft was able to function enough to return. Those airplanes that didn't return must have taken on critical hits. Therefore if we reinforce the areas not hit on the returning planes but presumed to be hit on the non-returning planes we should increase the number of returning planes.

3. We tackle this problem by saying that S which is the distance till achilles passes the tortoise is equal to : 10+5+2.5+1.25+0.625+…… this is an example of a geometric series. Without having to go into too much detail we can show the students a neat trick to solve this problem while introducing the concept of an infinite process. If we subtract 1/2(S)= 5+2.5+1.25+0.625…. from our original S we get the equation 1/2(S)=10 then we can solve for S=20. So we've established that there's a finite distance before Achilles is level with the tortoise. The paradox here is that we cannot ever take an infinite amount of steps so why should Achilles pass the tortoise however we also understand there's a finite distance before they should be level. In reality we know that Achilles passes the tortoise however in this problem we've introduced the idea of taking a finite distance and representing this distance as an infinite number of steps.(From this paradox discuss the concept of infinity with the students.)