Combinatorics
Problems with chessboards and paths

IMO 2002 Problem 1.
S is the set of all (h, k) with h, k non-negative integers such that $h+k < n$. Each element of S is colored red or blue, so that if (h, k) is red and $h' \leq h$, $k' \leq k$, then (h', k') is also red. A type 1 subset of S has n blue elements with different first member and a type 2 subset of S has n blue elements with different second member. Show that there are the same number of type 1 and type 2 subsets.

EGMO 2014 Problem 2.
A domino is a 2×1 or 1×2 tile. Determine in how many ways exactly n^2 dominoes can be placed without overlapping on a $2n \times 2n$ chessboard so that every 2×2 square contains at least two uncovered unit squares which lie in the same row or column.

IMO 2016 Problem 2.
Find all integers n for which each cell of $n \times n$ table can be filled with one of the letters I, M and O in such a way that: in each row and each column, one third of the entries are I, one third are M and one third are O; and in any diagonal, if the number of entries on the diagonal is a multiple of three, then one third of the entries are I, one third are M and one third are O. Note. The rows and columns of an $n \times n$ table are each labelled 1 to n in a natural order. Thus each cell corresponds to a pair of positive integer (i, j) with $1 \leq i, j \leq n$. For $n > 1$, the table has $4n - 2$ diagonals of two types. A diagonal of first type consists all cells (i, j) for which $i + j$ is a constant, and the diagonal of this second type consists all cells (i, j) for which $i - j$ is constant.

2014 Remote Training Problem 1.
Each of the twenty five 1×1 squares of a 8×8 table is coloured either red or blue. A spot is defined as a 1×1 blue square that shares an edge with a red square to its left or above it.

i) How many such 8×8 tables are there with no spots at all?

ii) How many such 8×8 tables have exactly one spot? (Hint: Split your non-spotted tables in suitable vertical areas and count all tables having a spot in one such area). Try to generalize your methods to an $n \times n$ table.