Mastermind.

- Mastermind is a game between a code-maker, and a code-breaker. The code-maker chooses a pattern of four colours. Duplicates are allowed, so the player could even choose four code pegs of the same colour. The chosen pattern is visible to the code-maker but not to the code-breaker.
- The code-breaker tries to guess the pattern, in both order and colour.
- Each guess is made by placing a row of code pegs on the decoding board. Once placed, the code-maker provides feedback by placing from zero to four key pegs in a separate space alongside the guess. A black key peg is placed for each colour guessed correctly and in the right position. A white peg indicates the existence of a correct colour placed in the wrong position.
- Play it here

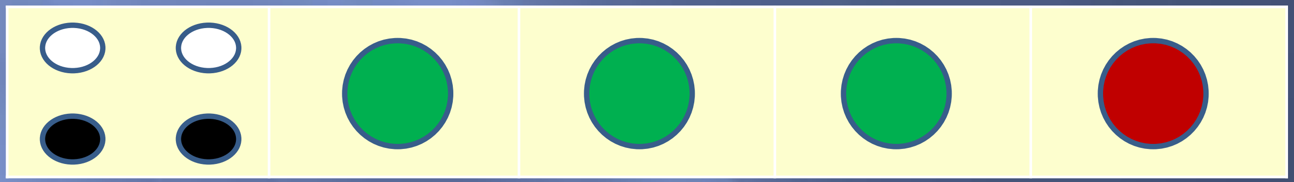
In each of the following examples, you made your first guess as a code-breaker and received feedback from the code-maker.

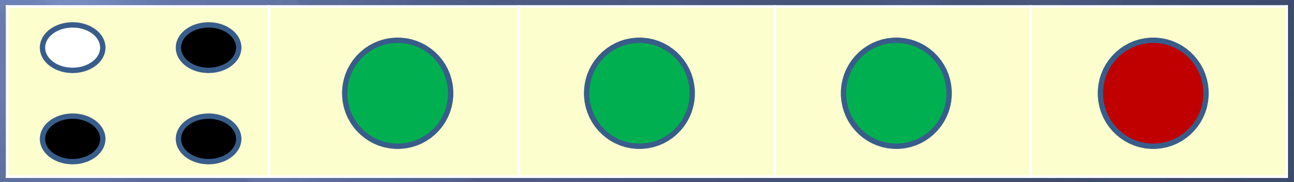
If your next guess takes thoroughly into account the information received so far, what are your chances that this second trial will be successful?

Sometimes the code-maker cheats by giving hints which are impossible.

Warm up

Two colours are in the right place, while two are not.

1 

2 

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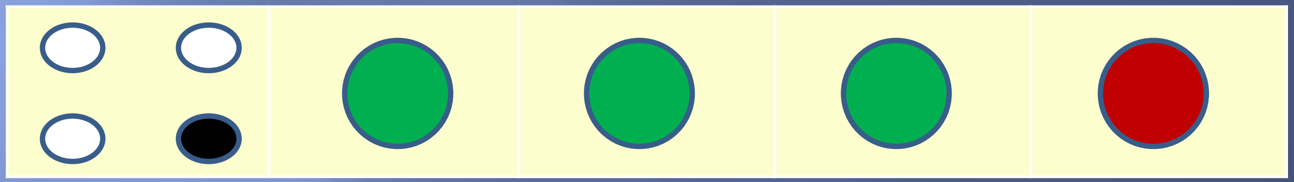

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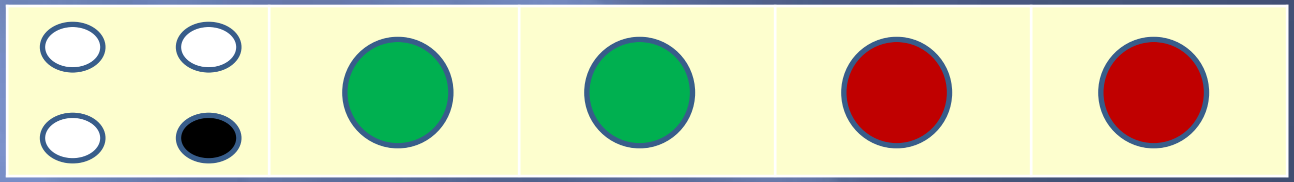

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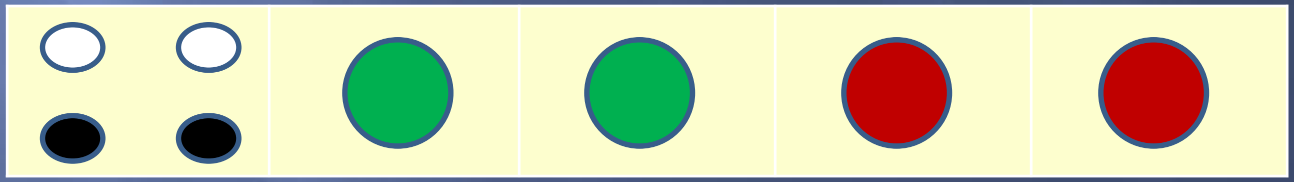

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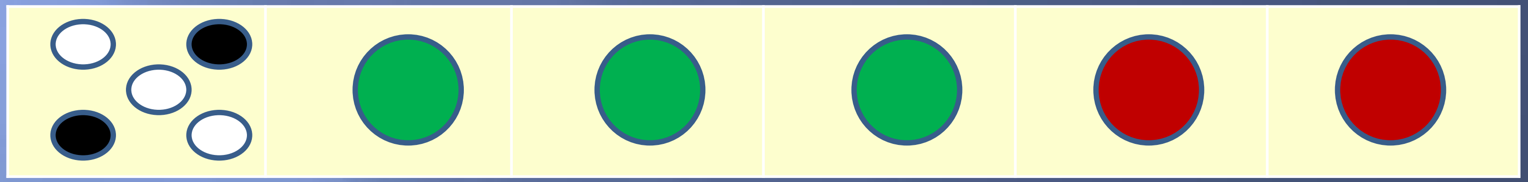

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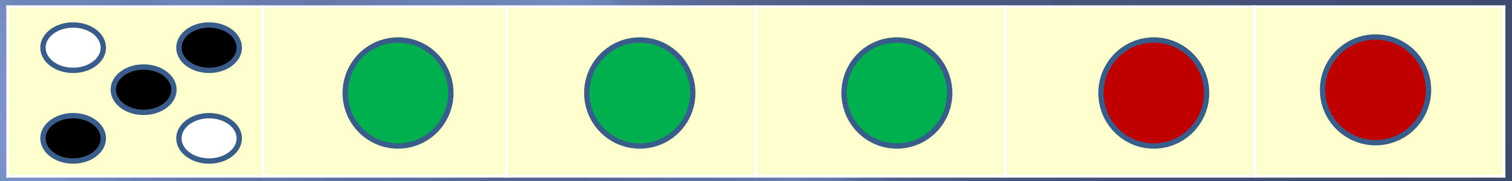

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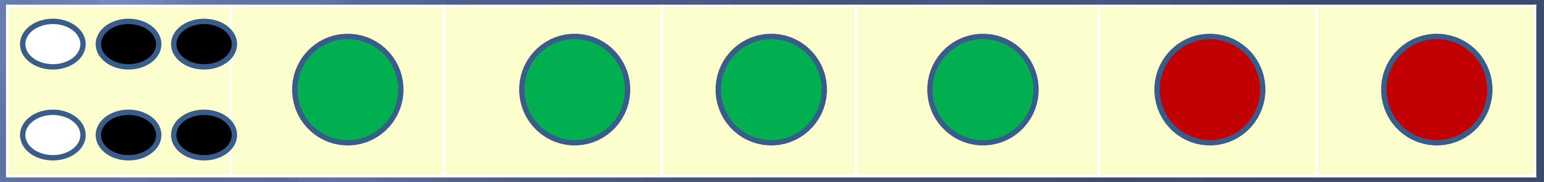


Arrangements=Permutations

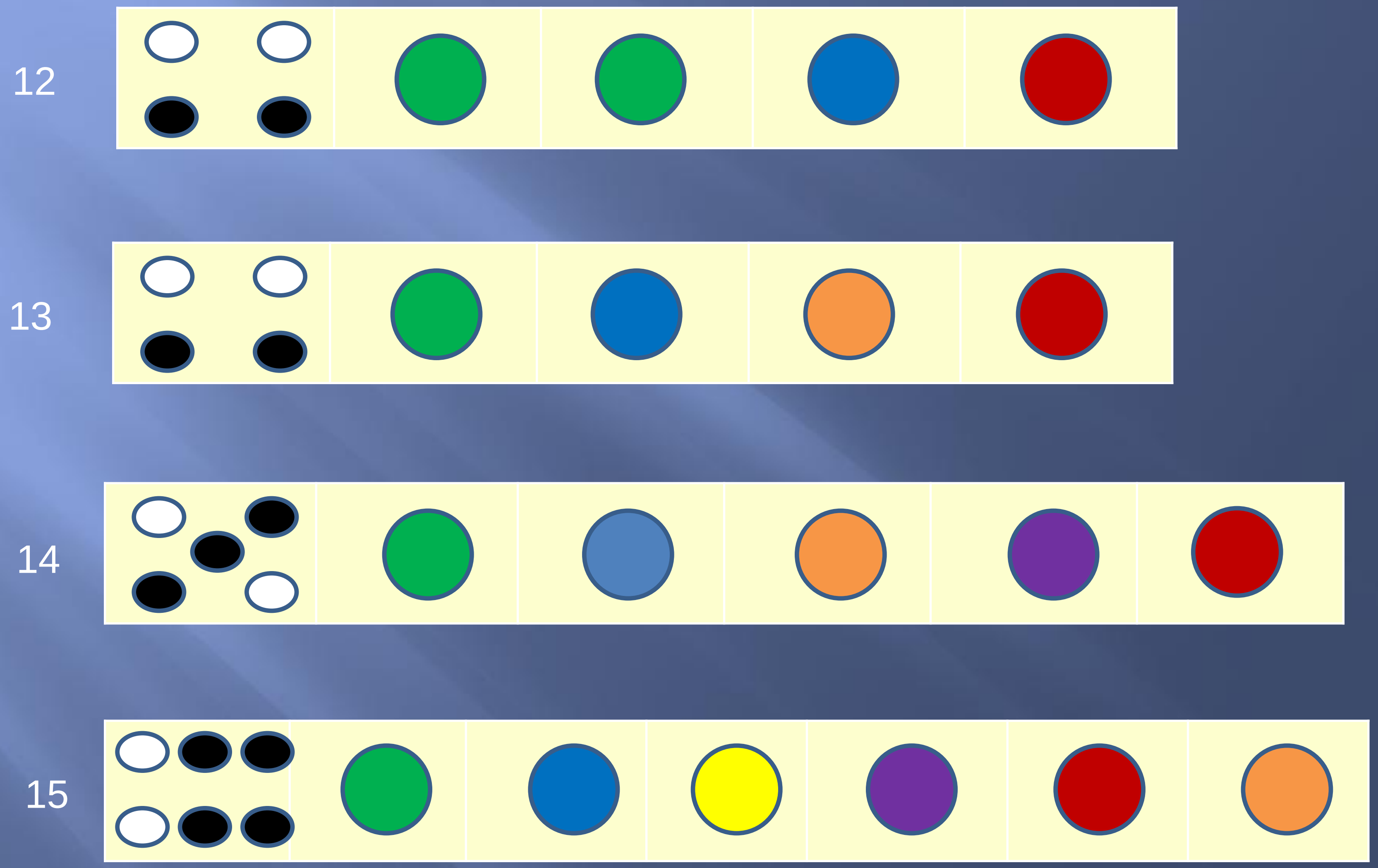
You are now the code-maker. There are 6 colours available. How many codes can you make:

9) In a game with 3 slots, if you don't repeat any colour.

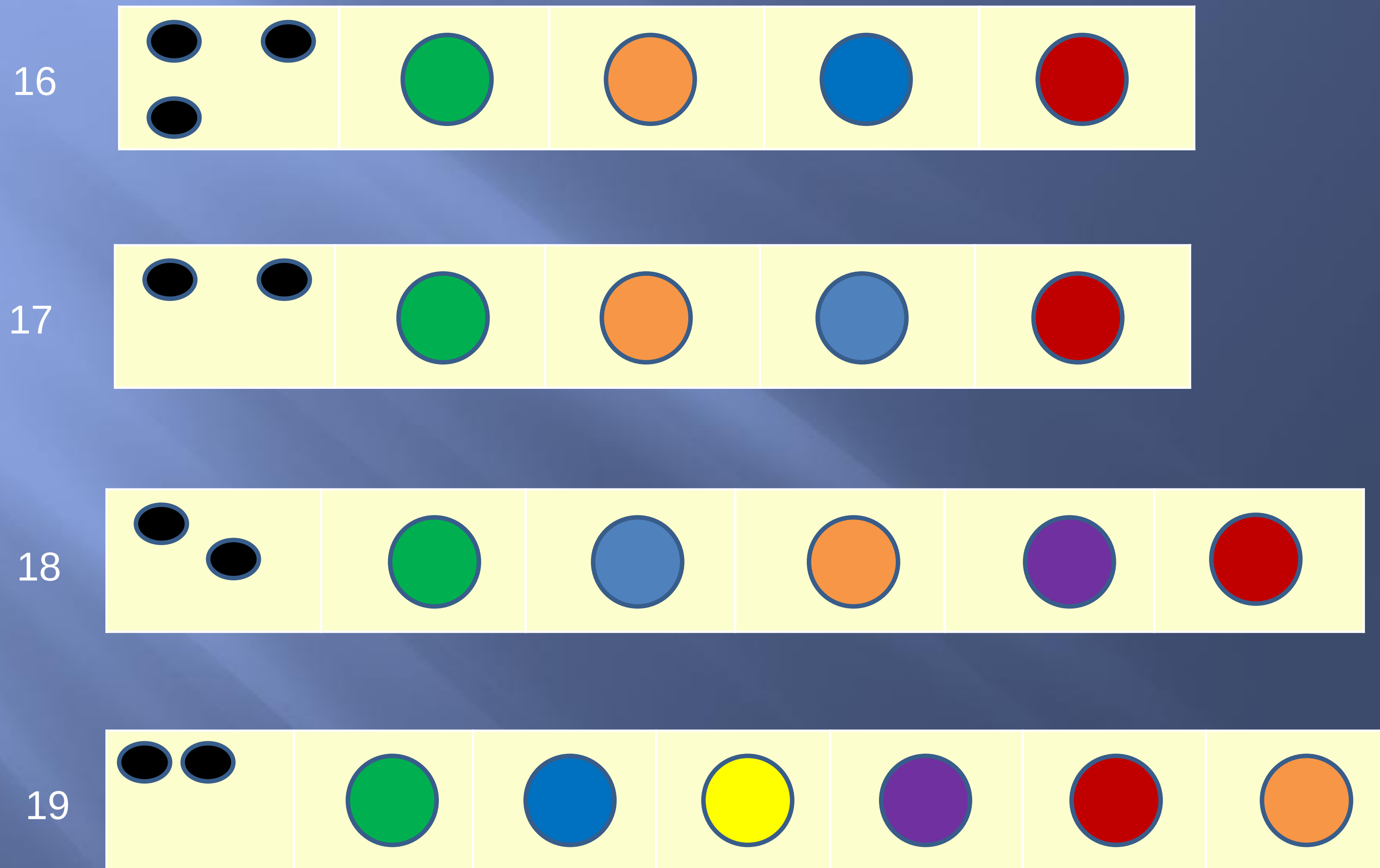
10) In a game with 4 slots, if you don't repeat any colour.

11) In a game with 5 slots, if you don't repeat any colour.

Combinations



All games have 6 colour choices.



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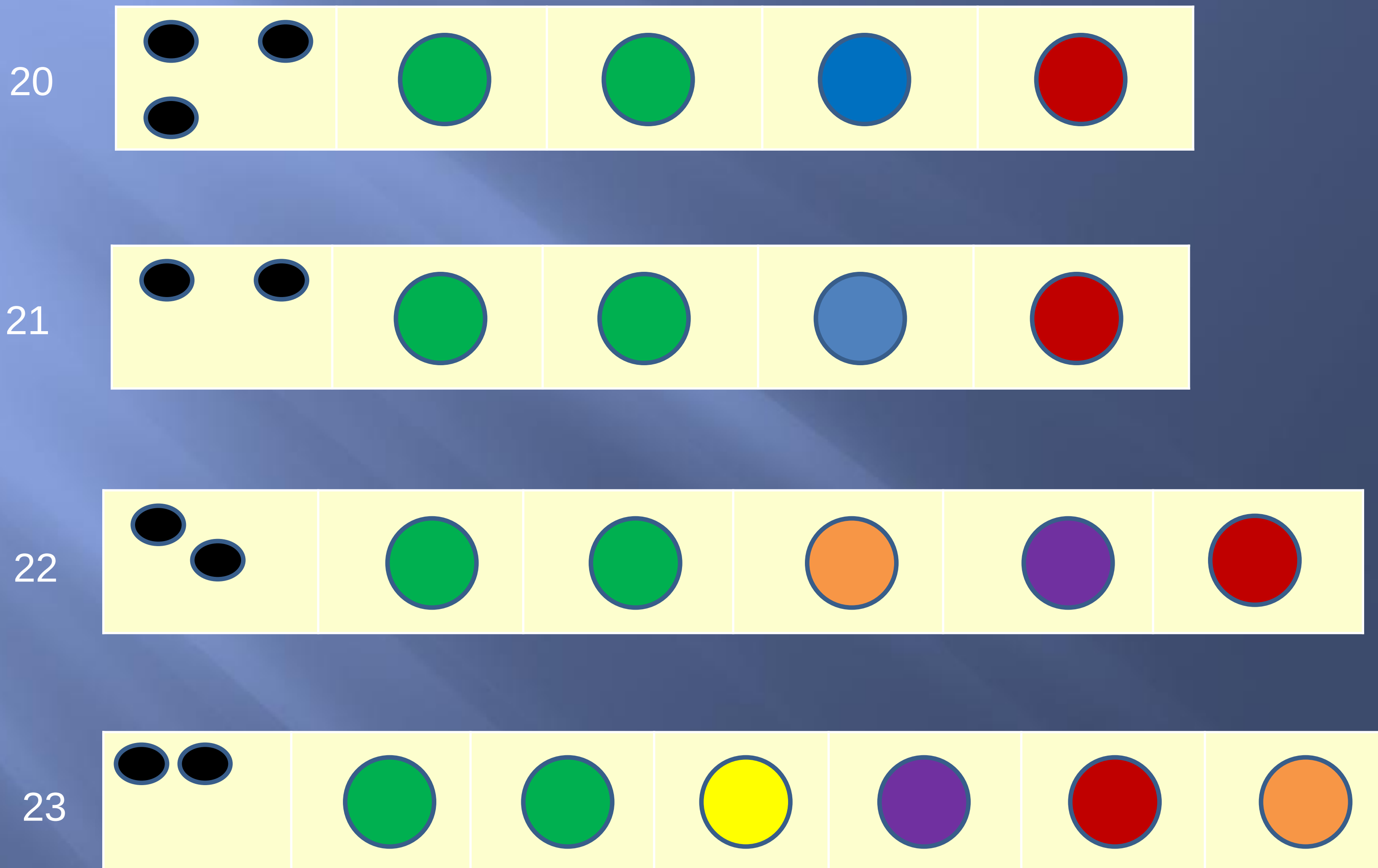


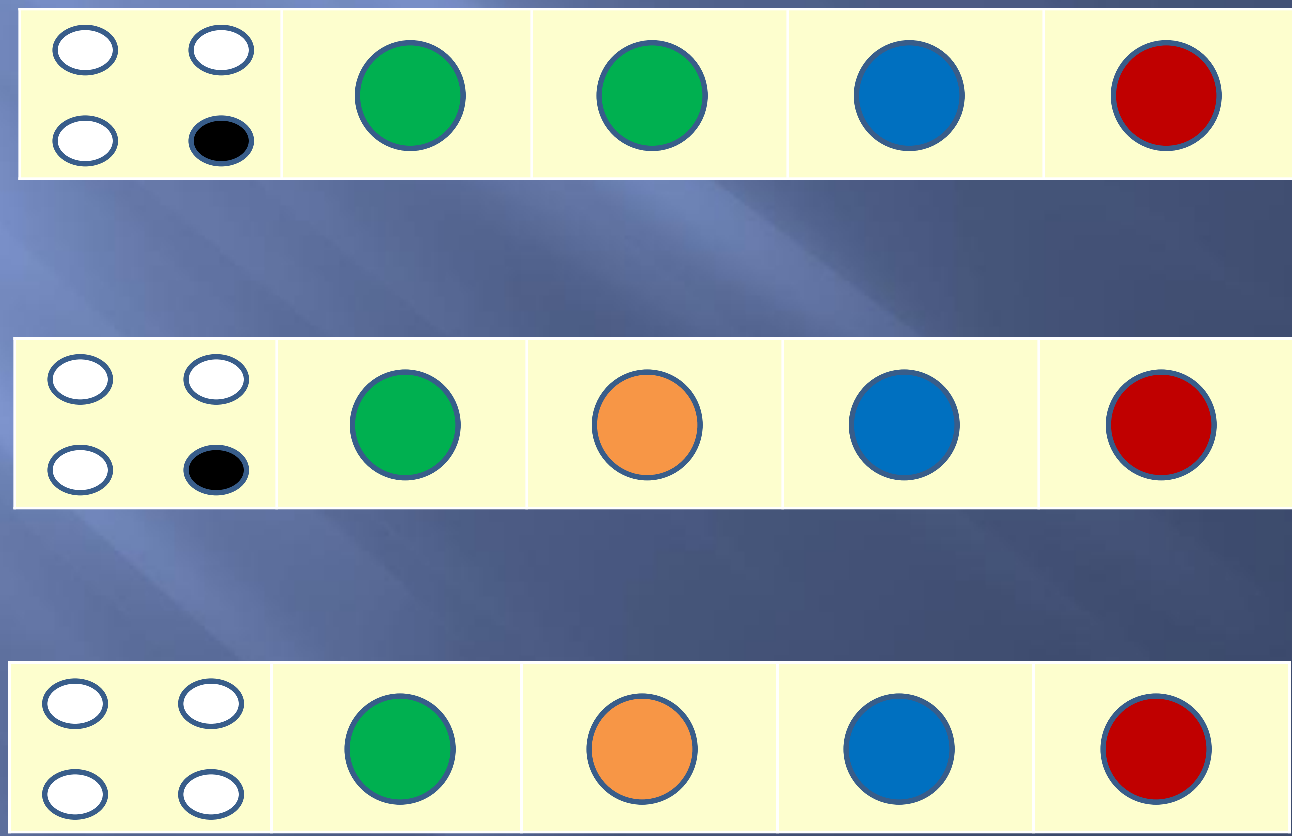
Products, Combinations

You are now the code-maker. There are 6 colours available. How many codes can you make:

24) In a game with 4 slots.
25) In a game with 5 slots if not all colours are the same.
26) In a game with 4 slots, if you wish your code to be made of exactly 2 out of the 6 colours.
27) In a game with 5 slots, if you wish your code to be made of exactly 2 out of the 6 colours.

Derrangements

You are code-breaker again. Count your chances of success in 1 move.



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Trivia?

- In 1977, the mathematician Donald Knuth, the father of the study of computer algorithms, demonstrated that the code-breaker can solve the pattern in five moves or less, using an algorithm that progressively reduced the number of possible patterns.

Links

- Wikipedia entry on Mastermind:
  http://en.wikipedia.org/wiki/Mastermind_(board_game)
- Play Mastermind online:

  http://www.puffgames.com/mastermind/

  http://www.web-games-online.com/mastermind/

- Mathworld entry on Mastermind with additional links:
  http://mathworld.wolfram.com/Mastermind.html
- UCC Enrichment information (Senior Cycle Students):

  http://euclid.ucc.ie/pages/MATHENR/index.html

  History and development of maths circles in the US:
  http://minerva.msri.org/files/circleinabox.pdf

Solutions

- 1) 1 in 3 chances. Red can be swapped with any green.
- 2) No chance -- the code-maker is cheating! 3 blacks+1 white =impossible.
- 3) Impossible – any switch will keep at least two peg colours fixed.
- 4) Impossible – any switch will keep at least two peg colours fixed.
- 5) 2x2=4 ways to switch red with green. So ¼ chances.
- 6) Impossible – only an even number of pegs can switch positions.
- 7) 2x3=6 ways to switch red with green. So 1/6 chances.
- 8) 2x4=8 ways to switch red with green. So 1/8 chances.
- 9) 6x5x4.
- 10) 6x5x4x3.
- 11) 6x5x4x3x2
- 12) 1/5 chances. Switch any green with blue or red, or switch blue with red.

- 13) 1/6. In how many ways can you choose a subset of 2 colours out of the four colours = (4 choose 2)= 3+2+1=6.
- 14) 1/10. (because (5 choose 2)= 4+3+2+1=10).
- 15) 1/15, because (6 choose 2)=15.
- 16) 1/20, because you choose the wrong peg (4 choices) and replace it with another colour (5 choices, since you exclude the wrong one).
- 17) 1/96, because you choose 2 wrong pegs out of 4 (6 choices), and replace each of them with another colour (4 choices each , since you exclude the 2 wrong ones).
- 18) 1/270, because you choose 3 wrong pegs out of 5 (10 choices), and replace each of them with another colour (3 choices each, since you exclude the 3 wrong ones).
- 19)1/240, because 240=15x2^4.
- 20)1/20 like in 13).
- 21) 1/(1x5^2+5x4^2).
- 22)1/(3x4^3+7x3^3).
- 23)1/(6x3^4+9x2^4).

- 24) 6^4.
- 25) 6^5.
- 26) (6 choose 2)x(4 choose 2)=90.
- 27) (6 choose 2)x (5 choose 2)=150.
- 28)1/4 (one of the greens must be fixed=2 choices, in each case permute the other pegs keeping none fixed=2 choices).
- 29)1/(4x(permutations of 3 pegs with no fixed peg))=1/8.
- 30)1/ (permutations of 4 pegs with no fixed peg)=1/9.

Permutations of 4 pegs with no fixed peg= 4x3x2-4x2-6-1=9.