

# UCC Mathematics Enrichment - Algebra

Kieran Cooney - kieran.cooney@umail.ucc.ie

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1. Calculate the value of

$$\frac{2014^4 + 4 \times 2013^4}{2013^2 + 4027^2} - \frac{2012^4 + 4 \times 2013^4}{2013^2 + 4025^2}$$

2. Find the value of

$$\frac{1^4 + 2007^4 + 2008^4}{1^2 + 2007^2 + 2008^2}$$

3. Find all solutions to the following system of equations:

$$x_1 + x_2 + x_3 = 1$$

$$x_2 + x_3 + x_4 = 1$$

...

$$x_{2014} + x_{2015} + x_1 = 1$$

$$x_{2015} + x_1 + x_2 = 1$$

4. Prove that

$$\sqrt[3]{20 + 14\sqrt{2}} + \sqrt[3]{20 - 14\sqrt{2}} = 4$$

5. Find all real solutions to the following system of equations:

$$ab + c + d = 3$$

$$bc + d + a = 5$$

$$cd + a + b = 2$$

$$da + b + c = 6$$

6. Find all polynomials

$$f(x) = a_0 + a_1x + \cdots + a_nx^n$$

satisfying the equation

$$f(x^2) = (f(x))^2$$

for all real numbers  $x$ .

7. Given that  $x, y, z$  are positive real numbers satisfying the following two equations,

$$x^2 + y^2 + z^2 = 25$$

$$xy + yz + zx = 18$$

evaluate  $x + y + z$ .

8. Solve the following equations for  $x$ :

(a)

$$x^3 + 3x^2 + 3x + 28 = 0$$

(b)

$$x^3 - 6x^2 + 12x + 1 = 0$$

(c)

$$27x^3 + 27x^2 + 9x + 9 = 0$$

9. The real numbers  $\alpha, \beta$  satisfy the equations

$$\alpha^3 - 3\alpha^2 + 5\alpha - 17 = 0$$

$$\beta^3 - 3\beta^2 + 5\beta - 11 = 0$$

Find  $\alpha + \beta$ .

10. Solve the following system of equations in positive real numbers:

$$(a + \frac{1}{a})(b + \frac{1}{b}) = 2(c + \frac{1}{c})$$

$$(b + \frac{1}{b})(c + \frac{1}{c}) = 2(a + \frac{1}{a})$$

$$(c + \frac{1}{c})(a + \frac{1}{a}) = 2(b + \frac{1}{b})$$