

UCC Mathematics Enrichment - Combinatorics

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Let \mathcal{S} be a finite set of at least two points in the plane. Assume that no three points of \mathcal{S} are collinear. A windmill is a process that starts with a line ℓ going through a single point $P \in \mathcal{S}$. The line rotates clockwise about the pivot P until the first time that the line meets some other point belonging to \mathcal{S} . This point, Q , takes over as the new pivot, and the line now rotates clockwise about Q , until it next meets a point of \mathcal{S} . This process continues indefinitely. Show that we can choose a point P in \mathcal{S} and a line ℓ going through P such that the resulting windmill uses each point of \mathcal{S} as a pivot infinitely many times.