

What you need: 2 jars, 20 marbles/coins

The game: You are presented with 2 empty jars and 20 marbles. There are 10 black marbles and 10 white marbles. You are to put all 20 of the marbles into the 2 jars in any way you choose. You then close your eyes and I will shake the jars up to ensure the marbles have been mixed properly. I will rearrange the jars on the table so you do not know which jar is which. You may then request either the left or right jar. You get to choose one marble from the jar. How do you maximise the probability of getting a white marble?

Strategy: The probability of selecting a white marble is maximised by placing one white marble in one jar and the remaining 19 marbles in the other jar. This increases the odds of picking a white marble from 0.5 to 0.725

The maths: You select the jar with the 1 marble with probability 0.5 and then select a white marble with probability 1.

You select the jar with the 19 marbles with probability 0.5 and a white marble with probability 0.45.

Adding together: $1 \times 0.5 + 0.45 \times 0.5 = 0.725$

Even if the students don't understand the probability involved in this problem they can still come up with this answer by intuition.

What you need: 2 dice

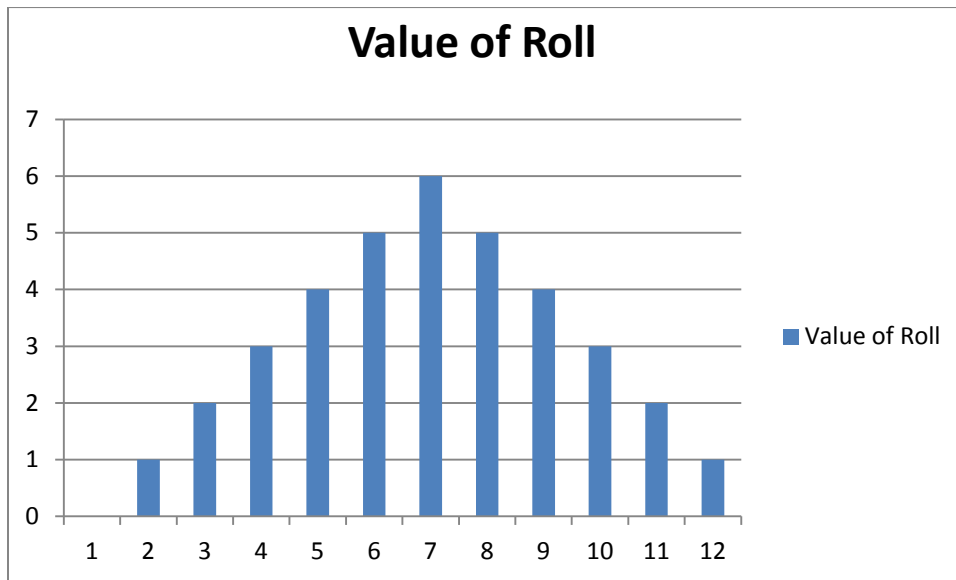
The game: The aim of the game is to guess what number will come up on the dice. You get a point every time you guess correctly. The first to 5 wins.

Strategy: Guessing 7 maximises your chances of winning.

The maths:

		<i>Roll 2</i>					
		1	2	3	4	5	6
<i>Roll 1</i>	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

7 is the most frequently occurring number.



The students could plot and understand this histogram.

Mastermind:

Setting Up: Mastermind Game

The Rules:

- One Player makes a code from the 5 coloured pegs and then covers it
- The other player must then attempt to guess the code
- For every peg that the second player gets the correct colour in the code but places in the wrong position the first player puts a white peg in the slots beside the code, for every peg that the second player gets the **correct colour** in the code and places in the **correct position** the first player puts a **black peg** in the slots beside the code
- The second player then makes another attempt at guessing the code using the information from the previous guess
- This is repeated until the player runs out of guesses or solves the code!

The Maths

Mastermind is a game based on combinations, permutations and probability, where the players must use logic and problem solving to break the code. Although the game can be easily played without understanding the maths behind it, it can be used to clearly explain combinations and permutations in a visual manner.

Further Questions

You are the code-maker. There are 5 colours available. How many codes can you make:

- In a game with 1 slot Ans: 5
- In a game with 2 slots with no repetition Ans: $5*4$
- In a game with 2 slots with repetition Ans: $5*5$
- In a game with 3 slots with no repetition of colours Ans: $5*4*3$
- In a game with 3 slots with repetition of colours Ans: $5*5*5$
- In a game with 4 slots with no repetition of colours Ans: $5*4*3*2$
- In a game with 4 slots with repetition of colours Ans: $5*5*5*5$
- In a game with 4 slots, if you wish your code to be made of exactly 2 out of the 5 colours.
Ans: $5C2*4C2$ where $5C2$ is the number of ways you can choose the colours and $4C2$ is the number of ways you can arrange the 2 colours

In a game with 5 slots, if you wish your code to be made of exactly 2 out of the 5 colours.

Ans: $5C2*5C2$

Other combinatorics questions

At a wedding how many ways can 6 people be arranged for a photograph if the bride must be next to the groom?

Ans: $5! \cdot 2$ (Think of the bride and groom as 1 unit).

At a wedding how many ways can 6 people be arranged for a photograph if the bride must be to the left of the groom?

Ans: $6!/2$ (Half of the total amount of permutations, on the other half the bride will be to the right of the groom).

5 co-workers, part of a car pool drive to work. There are two seats in the front including the driver's seat and three at the back. If John or Kevin who are a part of this pool have to drive the car, then in how many ways can the 5 be seated in the car?

Ans: $4! \cdot 2$ (Consider the two cases where John and Kevin are driving individually and then add together).