Logic Puzzles

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1. **two guard riddle**
You face two guards: A knight and a knave. The knight will always tell the truth and the knave will always lie, and you do not know which is which. You must find out, which one is the knight and which one is the knave, and are only allowed to ask one question to one of the guards.

What question should you ask?

Answer: You should ask one of them, "If I asked you were you the knight, what would you say?"
If you are talking to the knight, obviously he will say "yes".
If you are speaking to the knave, had you asked him if he was the knight he would lie and say "yes"- because you are asking him what he would say, he will lie about his answer and so tell you that he is not the knight.

2. **three guard riddle:**
The three guard riddle is a more advanced problem:
You face three guards: Again a knight and a knave but also a joker who will tell you a random answer each time to try to confuse you! Also, these guards are from a far away land and use the words "Bal" and "Dal", for "yes" and "no" but you dont know which means "yes" and which means "no". We are allowed to ask 3 questions, and each guard will answer each question, and we must find out which guard is the knight, the knave and the joker.

Solution:

- **Step 1.** It would be nice to know how to translate Bal and Dal. So you ask a question which will surely elicit at least two Yes answers among the 3 guards. E.g. "Are you the knight?" Whichever of the words Bal or Dal comes up at least twice, thats a Yes!
- **Step 2.** You'll try to find out the knight/knave like you did in the old 2 guard problem. The joker can try to pass as a knight or the knave, but he cant imitate both at once. Thus by asking "If I asked you if you are the knight, what would you answer?" Whoevers answer is different from the other two, youll know who he is!
- **Step 3.** Now you know one guard whose answers you can rely on to find out the truth. E.g. if you found out who the knave is in Step 2, just negate all his answers and youll know the truth. So now you ask, pointing at one of the knight/joker pair: "Is this guy here a joker?"

3. **Numbers-Letter Cards**

In this exercise, four cards are laid out on a table. Each card has a letter on one side and a number on the other side. The sides of the cards that we can see read: “7”, “S”,“5” and “J”.

An unreliable source told us that whenever a “7” is on one side of a card, an “S” is on the other side. The task of this puzzle, is to check if this unreliable source is telling the truth. However, you can only turn over two cards. So, which two cards should you turn over?

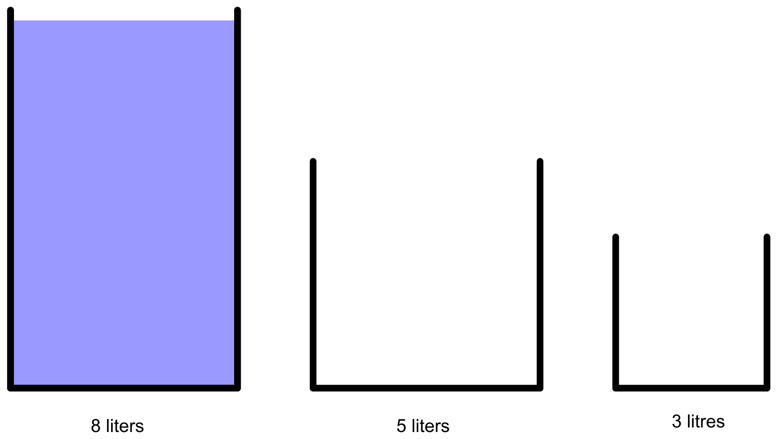
Answer: Turn over the “7” and the “J”.

Maths in the puzzle

The purpose of this exercise is to examine results of logic.

Suppose we know that A implies B. Then:

- B does not imply A.
- *not* A does not imply B.
- *not* B implies *not* A.



4. **Water container puzzle** You are given 3 water containers- One holds 8 liters and is full of water. The others hold 5 liters and 3 liters and they are empty. All three containers are unmarked.

   The object of the puzzle is to divide the 8 liters 2 parts of 4 liters each. The difficulty in this problem, is that we can not measure what one liter is. We have a measure of 3 liters and 5 liters (knowing 8 liters is useless, as this is all the water)

   solution:

8	0	0
5	0	3
5	3	0
2	3	3
2	5	1
7	0	1
7	1	0
4	1	3
4	4	0

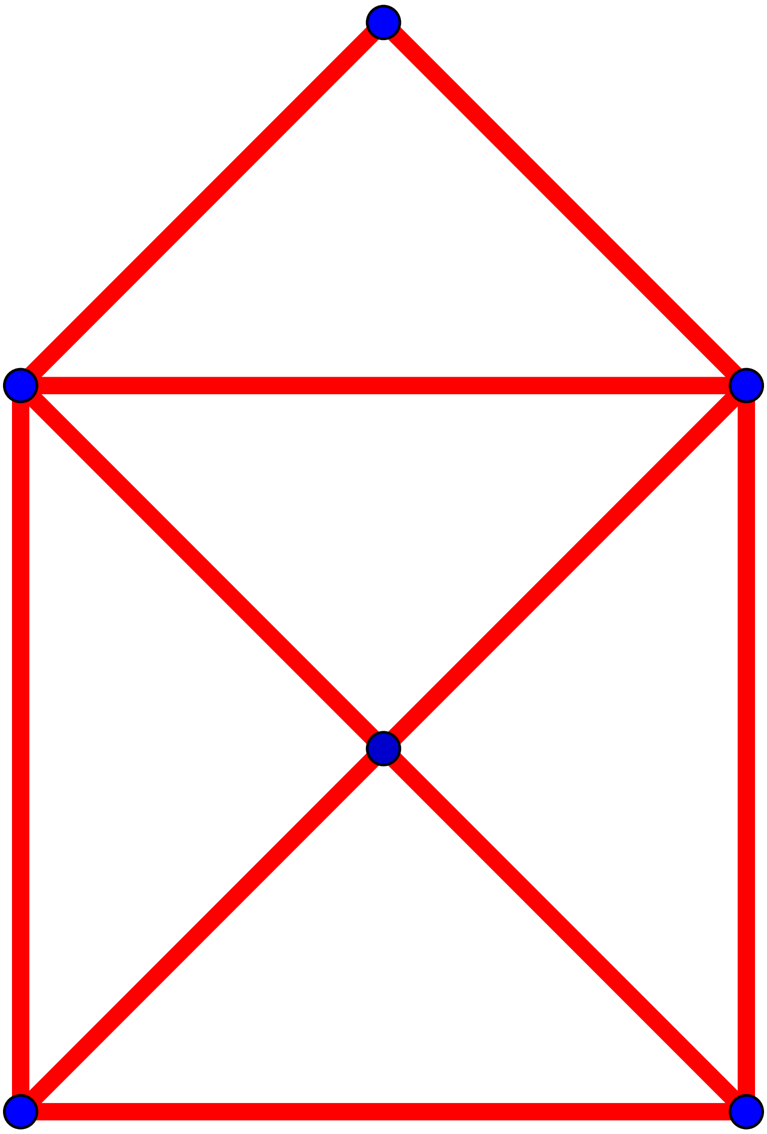
or:

8	0	0
3	5	0
3	2	3
6	2	0
6	0	2
1	5	2
1	4	3
4	4	0

Now suppose you have 3 containers, that hold 8 liters, 4 liters and 2 liters, again, with the 8 liter container being full and the other two empty. Try to divide the water into 2 parts- 5 liters and 3 liters.

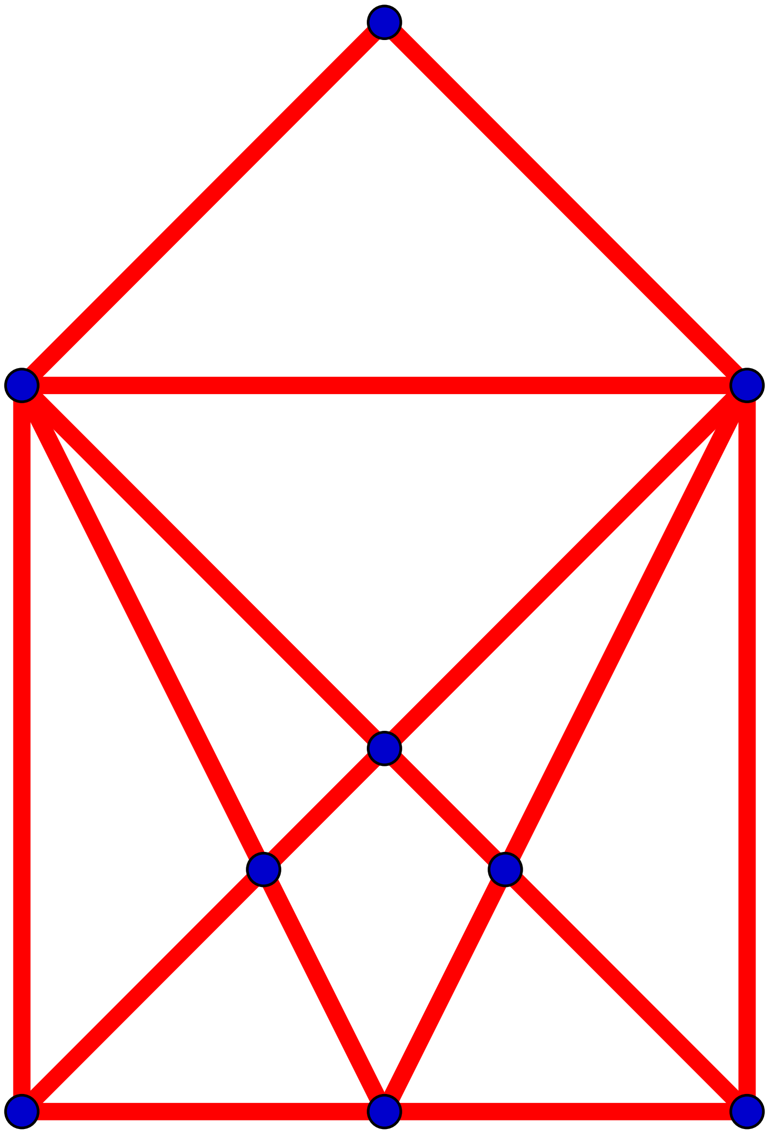
This is impossible! Because each of the containers hold an even number of liters, when we transfer water between containers we are just adding and subtracting even numbers, and so the result is always even, so we can not get an odd number of liters, such as 3 or 5.

5. **Eulerian Graphs**
Shown Below is the map of the roads connecting a number of towns. The blue dots are the towns and the red lines are the roads.



Create a path that travels over each road, once and only once.

Now, new towns have been developed and more roads had to be built:



Can you find a similar path in the new network of roads?

Maths behind the puzzle

(a) We look at each of the towns and count the number of roads touching it.

(b) If we have an even number then for every time we drive into the town, there will be a road for us to leave on.

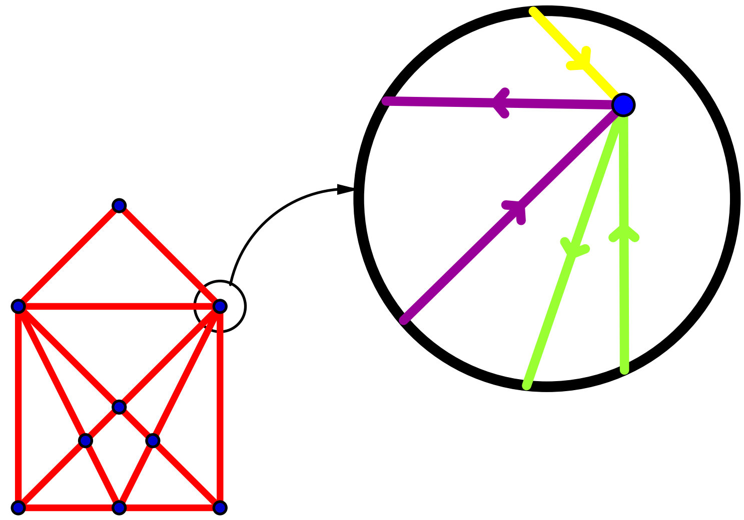


Figure 1: if we enter and exit by the green and purple roads first, when we then enter by the yellow road, there is no road left for us to exit by.

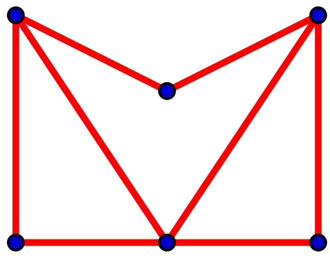
(c) If there is an odd number, then we must either start at that town but not finish there, or else finish there but not start there.

(d) So for there to be a possible path, there must either be no towns with an odd number of roads, in which case we start and finish at the same town, or else exactly two towns with an odd number of roads, in which case we start at one and end at the other.

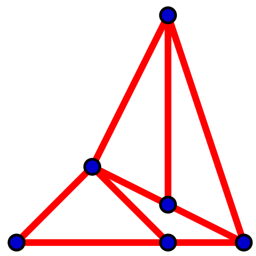
Graphs that have this property are called Eulerian graphs, named after Leonhard Euler, who discovered them.

Further Questions

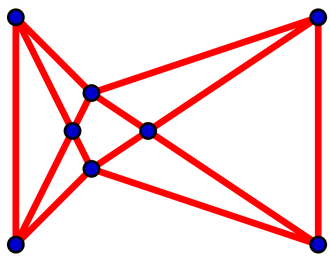
Which of the following are Eulerian graphs?



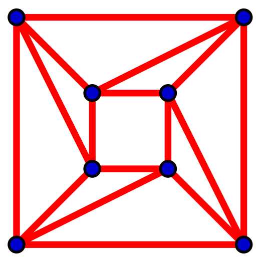
(a)



(b)



(c)



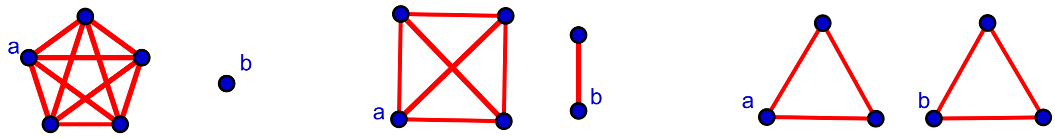
(d)

6. There are 11 roads between 6 towns. No two roads connect the same two towns. Show that one can use these roads to travel between any two towns.

Solution: To prove this, we use a technique called proof by contradiction- we assume that we can arrange the roads in such a way that it is impossible to travel from one town to another, and hope to arrive at a contradiction. In order for it to be impossible to move from town a to town b,, we must be able to separate the graph of roads and towns into two disjoint graphs, such that town a is in one graph and town b is in the other: There are three possibilities:

$$(a)\text{5 towns in one graph, 1 in the other}$$
$$(b)\text{4 towns in one graph, 2 in the other}$$
$$(c)\text{3 towns in both graphs}$$

In each of the cases, we can not draw the 11 roads between the two graphs,



(a) (b) (c)

and so we will need to connect the two graphs by a road, thus giving us a contradiction, that it is possible to build the roads so that we can not travel from town a to town b.