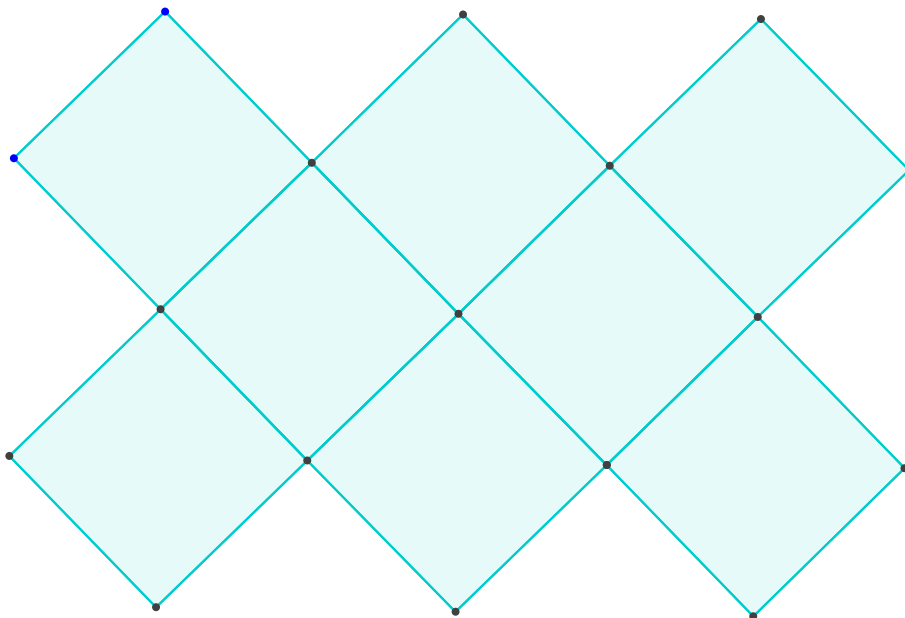
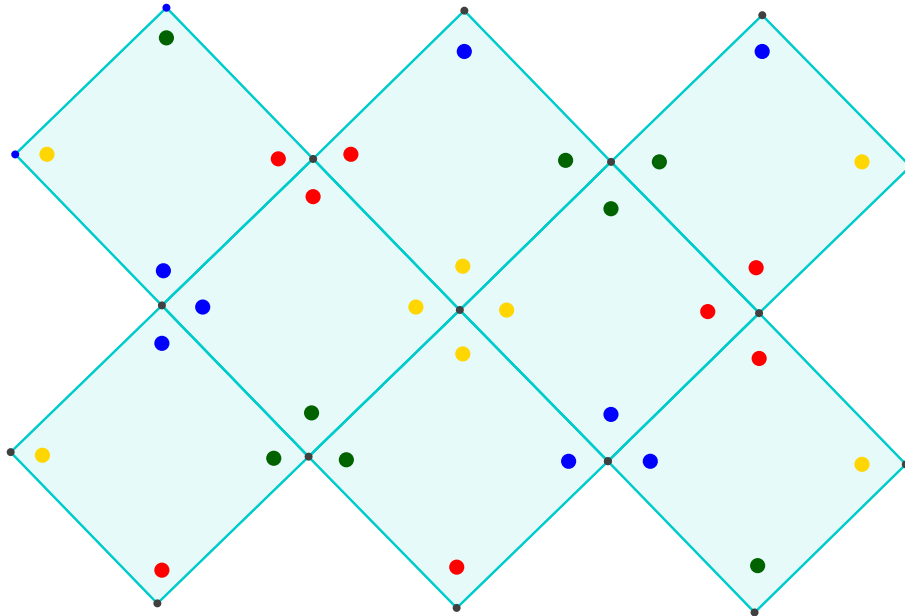
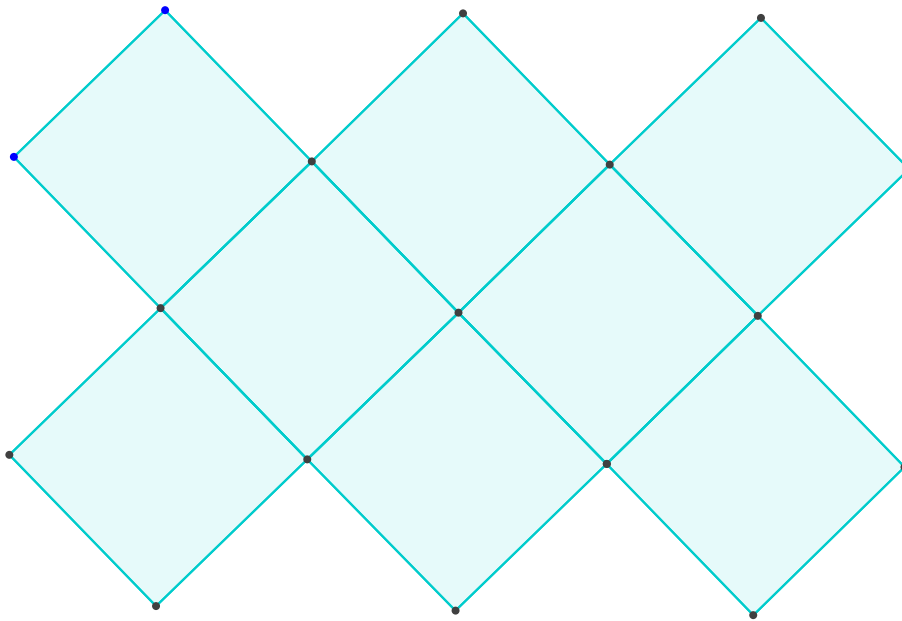


Diamond puzzle

Set-up: (For organizers) Print and cut out these squares, then arrange them randomly on the board below.



The goal is to rearrange the given pieces on the board such that each vertex has only one color around it.



Hints:

- (1) How many different types of pieces can you find? (To compare them, you can place all pieces by the board with the red circle in the upper right corner).
- (2) How many pieces of each type have you found?
- (3) Start filling in the puzzle with the two interior squares. Which pieces should you put in here, so that the remaining pieces are as varied as possible? (Place the two different types of more common pieces in the middle).
- (4) What happens if you rotate your initial piece on the board?

Extra questions:

How many possible ways to arrange the four colors on the vertices of a square are there?

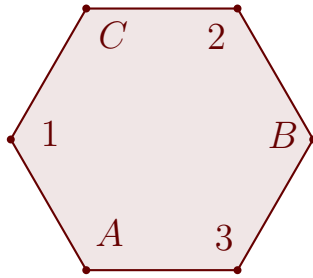
How many possible solutions does the puzzle have?

Solutions:

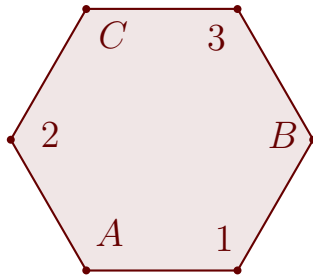
- (1) There are six different ways to color a piece with the given 4 colors. To see this, we fix the red circle is in the upper right corner. Then we have three color choices for the next corner, two for the next one, and one for the last. There are six types of colored squares.
- (2) There are 8 pieces, so there exist two pairs of pieces of the same type.
- (3) Choose one piece each from the common pairs and place them in the interior. In this way all types of pieces are available for the outer squares, which allows you to fill in the puzzle no matter in which position you places your first piece.
- (4) The puzzle works for each of the 4 possible initial positions of the initial square placed on the table, and as well if we flip the squares in the middle position.

Indeed, the two inner squares impose different constraints on the sides of each of the neighboring squares. Together, these conditions add up to all possible pairs of colors on the sides of the six types of colored squares.

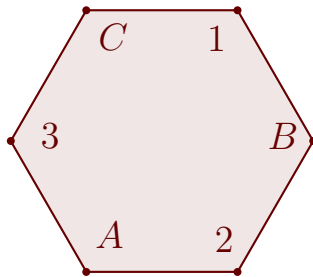
Small beehive puzzle**Set-up:** (For organizers) Print and cut out these shapes:



1 copy of this.

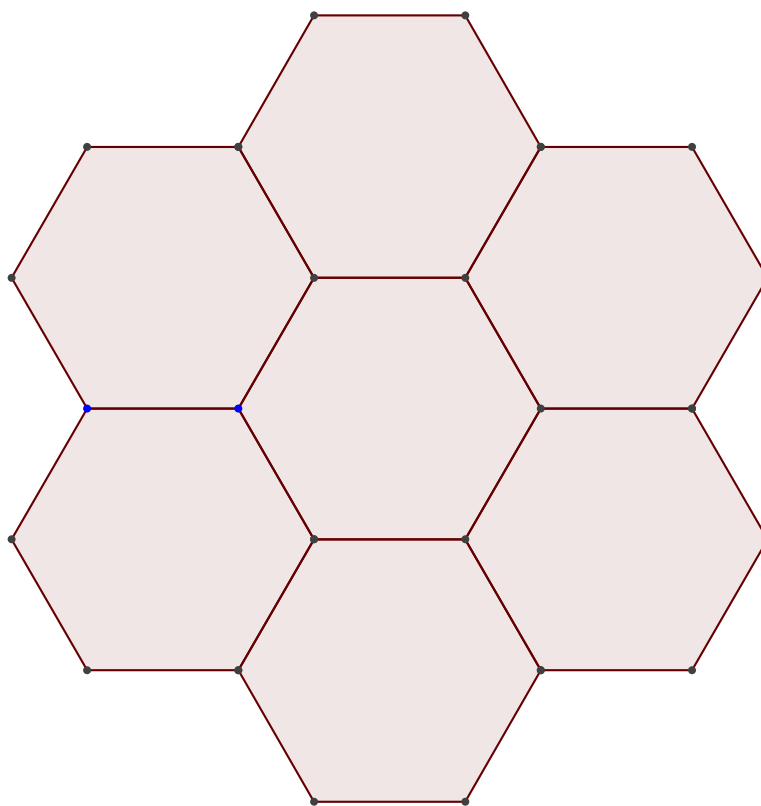


3 copies of this.



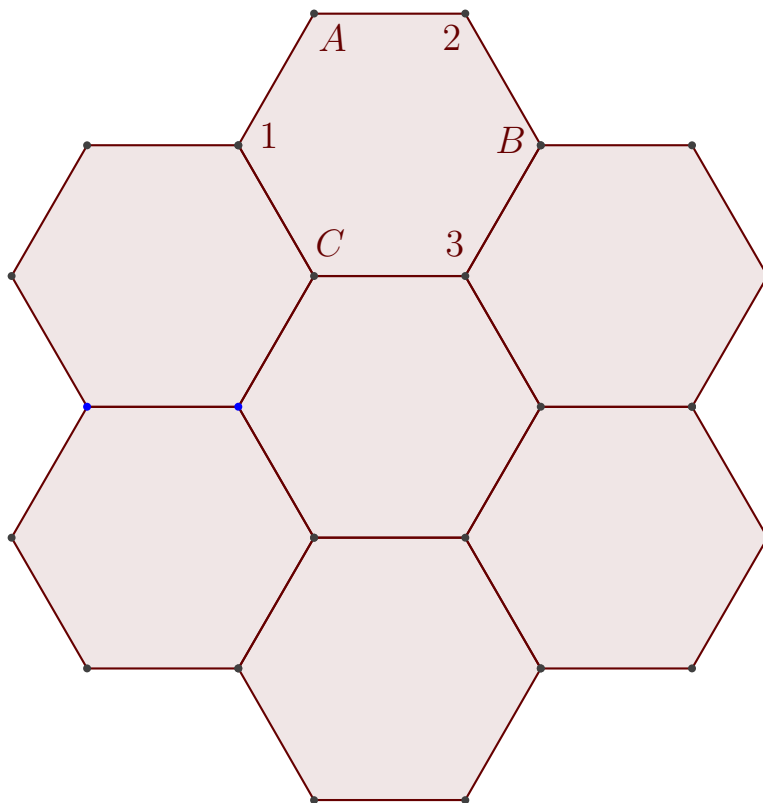
3 copies of this.

If you'd like, place the pieces randomly on the board:



The final goal of the puzzle is to rearrange the given pieces on the board such that each vertex in the beehive has one symbol written around it thrice, and also to find out how many different solutions there are. This can be presented as a sequence of 4 smaller challenges.

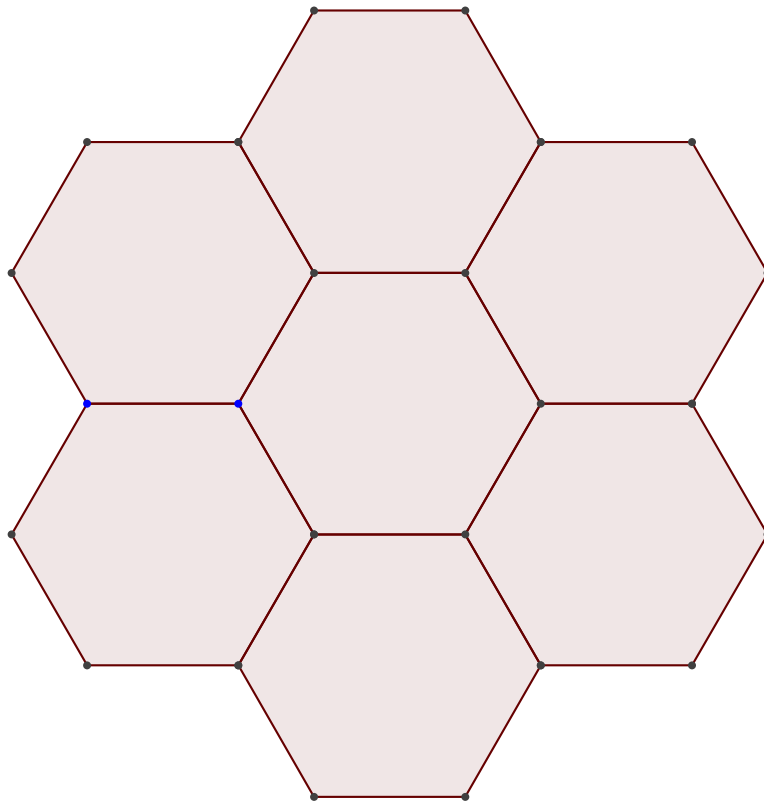
1. Label the vertices of each hexagon in the beehive after the model at the top, such that
 - Each hexagon contains the symbols A, B, C and $1, 2, 3$, and each of its sides has a letter at one end, and a number at the other end.
 - Each vertex has just one symbol written around it thrice.



- a) Is the solution unique? Explain why.
- b) How many types of labeled hexagons have you used? (If one labeled hexagon can be rotated so that it looks identical to another, then they are of the same type. For example, one type reads clockwise as $C - 1 - B - 2 - A - 3$). How many hexagons of each type are there?
- c) Where are all the hexagons of the same type distributed on the board?

2. a) Fill in the empty board below with the 7 pieces provided, so that each vertex in the beehive has one symbol written around it thrice.

b) How many different solutions are there?

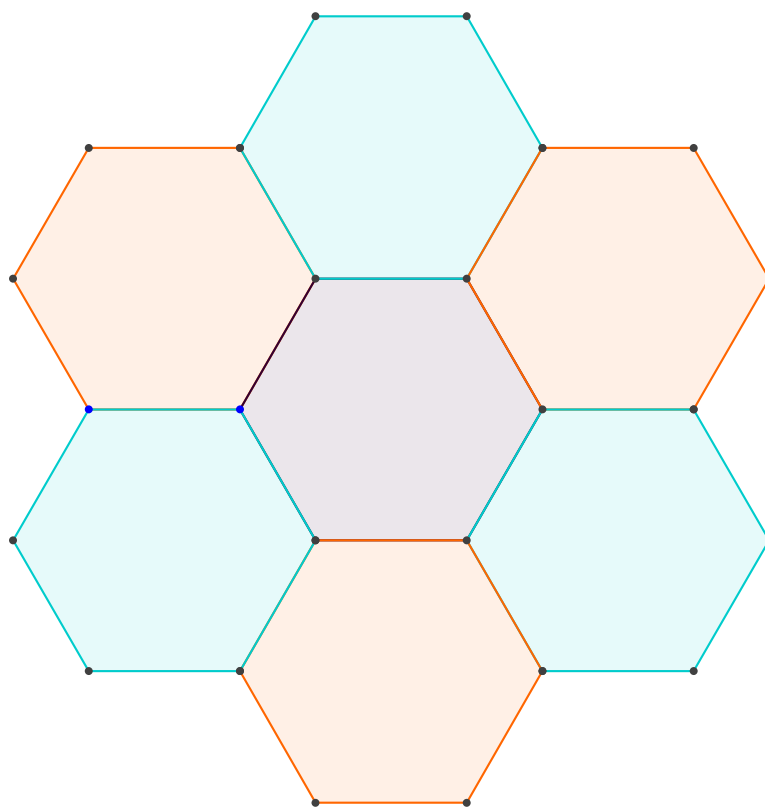


Solutions:

1. a) The two conditions listed lead to a unique solution, because the edges of the given labeled hexagon should be labeled in the same way in the neighboring hexagons, and the other vertices in these hexagons are determined by the requirement that the same number/letter cannot appear twice in the same hexagon.

b) There are three types, read clockwise as: $C-1-B-2-A-3$ and $C-2-B-3-A-1$ and $C-3-B-1-A-2$.

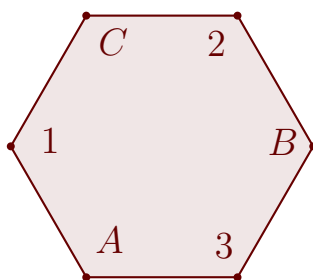
c) The hexagons of the same type are found in places of the same color:



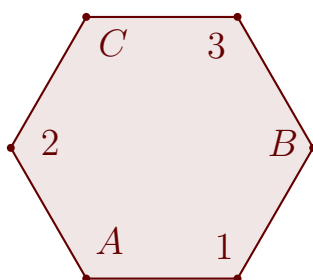
2. The number of pieces of each type tells us which piece to place in the center of the beehive. There are 6 different ways in which we can place it, due to possible 60° rotations. In each case, there is a unique way to fill in the puzzle.

Large beehive puzzle

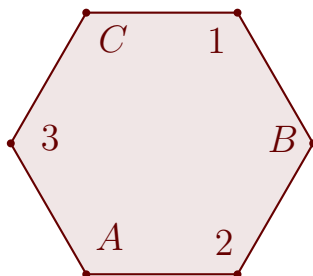
Set-up: Print and cut out these shapes:



4 copies of this.

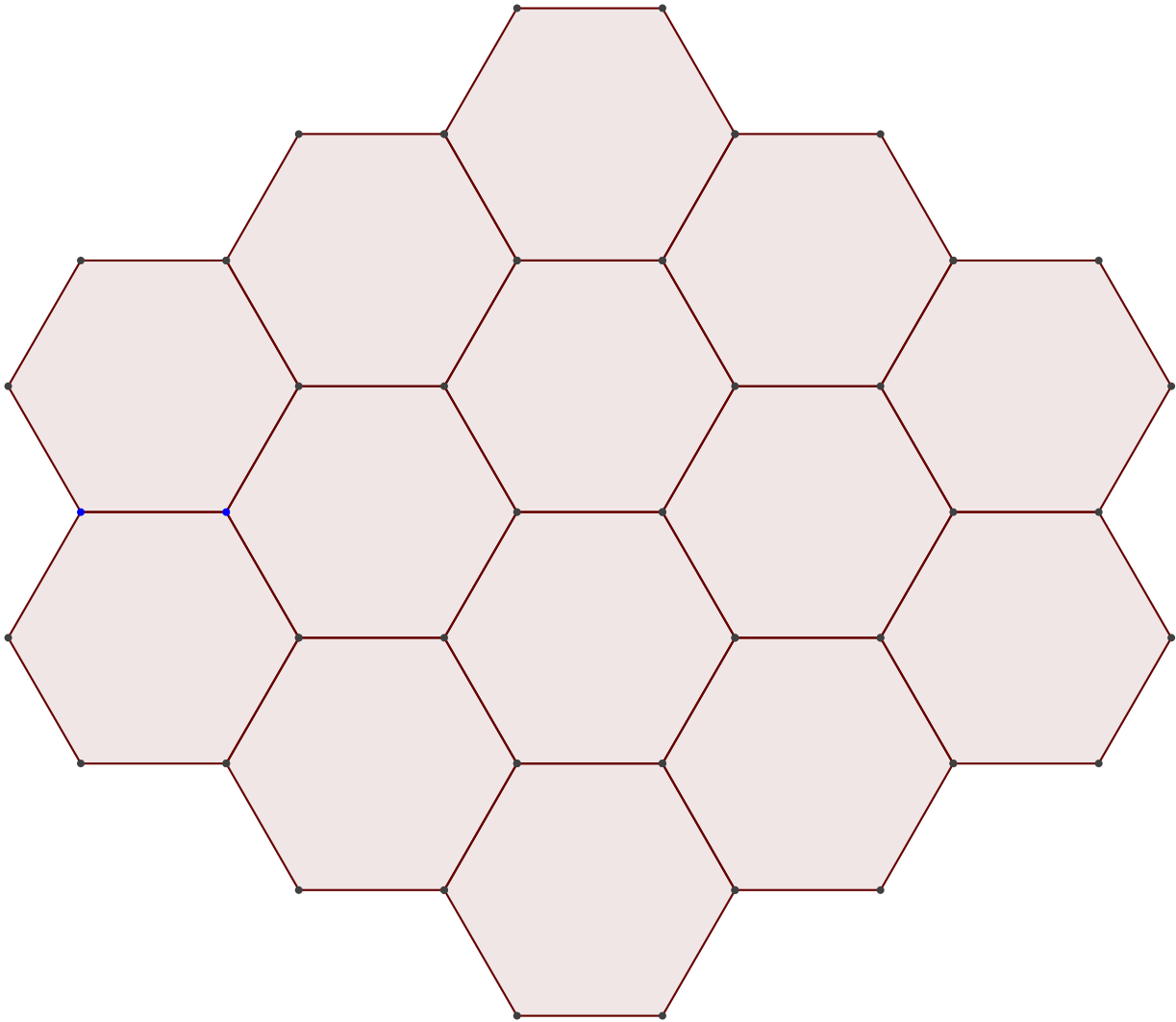


5 copies of this.



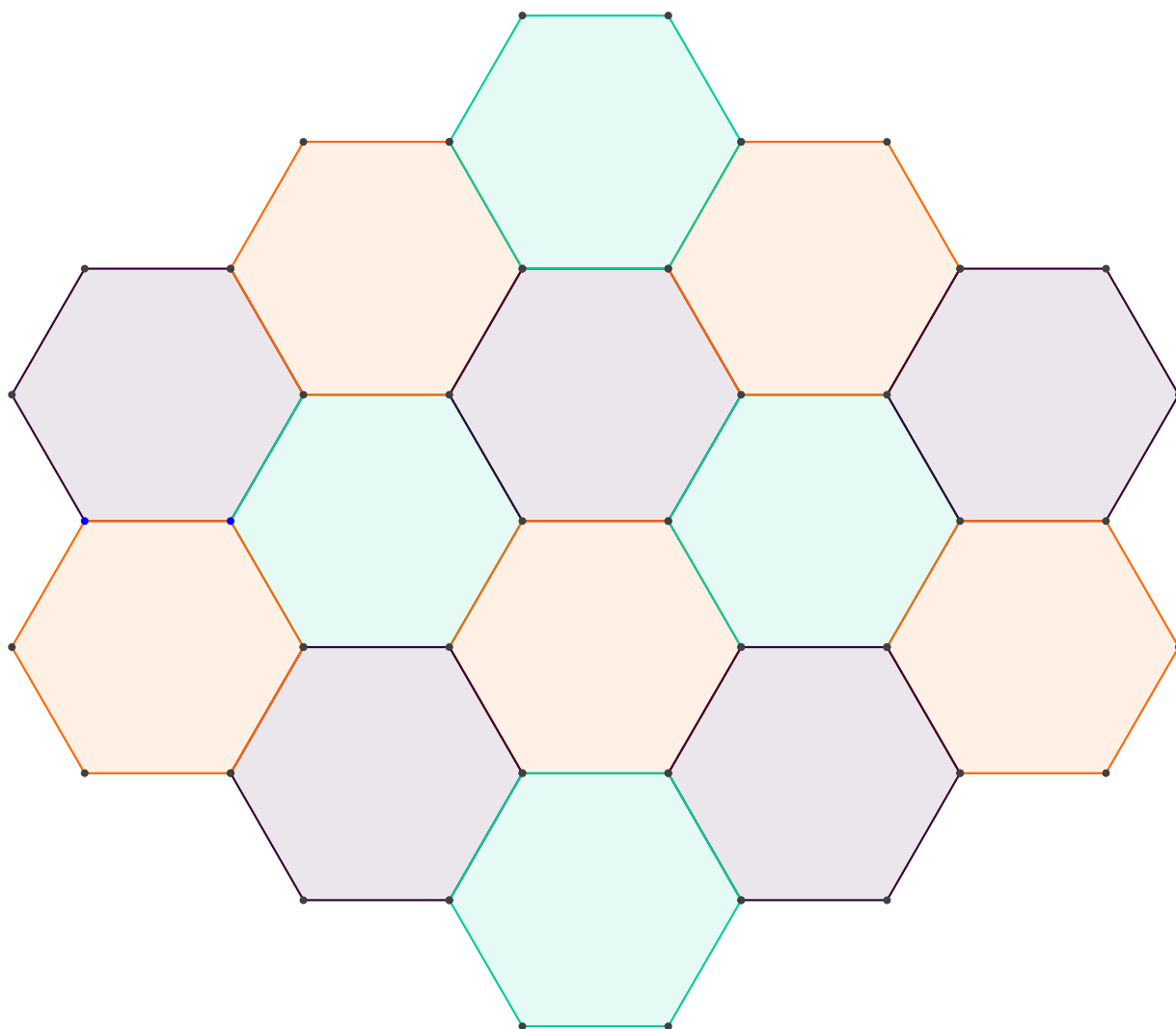
5 copies of this.

Fill in the empty board below with the 7 pieces provided, so that each vertex in the beehive has one symbol written around it thrice. How many different solutions are there?



Solution

The three different types of hexagons should be placed as follows: (each color corresponds to one type). There are 6 solutions again, depending on how we rotate the first piece we place on the board.



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