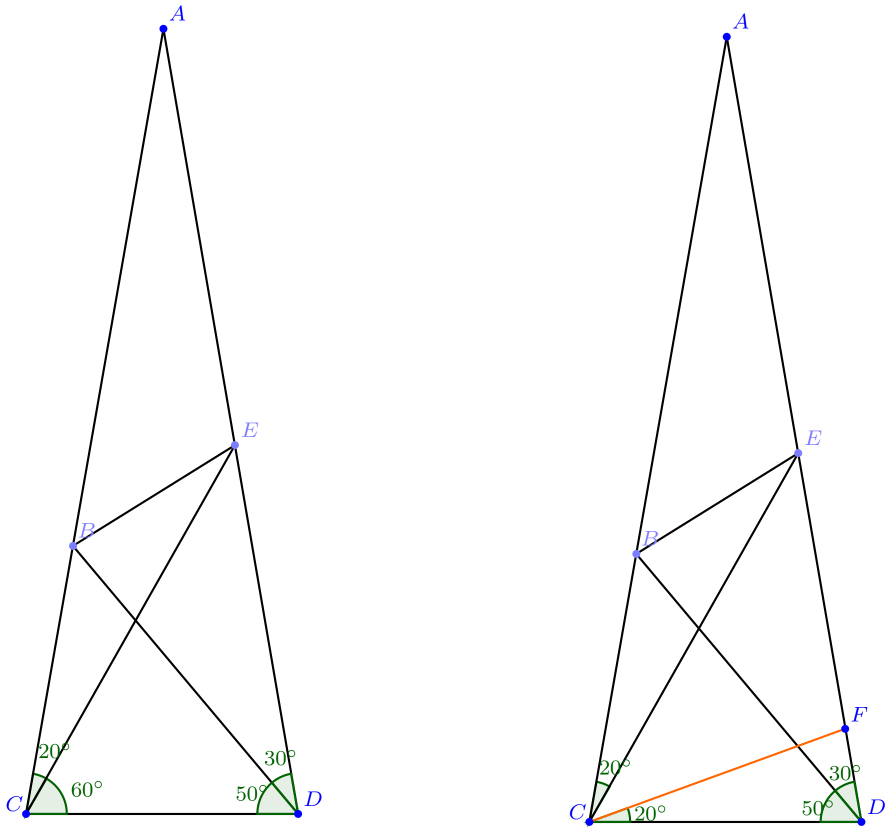
Mahatma's triangle

Traditional methods for solving missing angle problems have always been about finding out as many angles as you can and see if you can stumble upon a solution. This particular problem looks deceptively easy, with one large triangle and five smaller inner triangles. The goal is to find the angle at CÊB.



Hints. Let's break the problem into a sequence of smaller challenges:

- In the left hand side figure, find all the angles you can calculate. How many isosceles triangles can you find? Recall that for a triangle to be isosceles, it is enough that the angles at the base of the triangle be equal.
- Now look at the right hand side figure. Here we drew an extra line CF, so that FĈD= 20°. How many isosceles triangles can you find now? Can you find any equilateral triangle?
- Can you now find the angle at CÊB?

Full Solution.

Add a point F on the line AD so F$\hat{\text{C}}$D= 20°.

By construction, $\triangle FCD$ is isosceles with C$\hat{\text{F}}$D= 80°, F$\hat{\text{C}}$D = 20° and F$\hat{\text{C}}$E = 40°.

From this we can look at $\triangle BCD$ and $\triangle CED$ and find angles C$\hat{\text{B}}$D =50°and C$\hat{\text{E}}$D =40°.

From this, we can see that $\triangle BCD$ is isosceles with CD = CB.

We can then use that fact that CF = CD so CF = CB which makes $\triangle CFB$ also isosceles so C$\hat{\text{B}}$F = C$\hat{\text{F}}$B = $\frac{180-BCF}{2}$ = $\frac{180-60}{2}$ =60°.

Which makes $\triangle CFB$ equilateral as CF = CB = FB.

This allows us to work out B$\hat{\text{F}}$E using $180 - 60 - 80 = 40°$.

Noting that F$\hat{\text{C}}$E = F$\hat{\text{E}}$C so CF = EF and from before CF = FB which means FB = EF making $\triangle BFE$ isosceles. As before $\frac{180-EFB}{2}$= $\frac{180-40}{2}$=70°.

This gives us F$\hat{\text{B}}$E = F$\hat{\text{E}}$B = 70° so B$\hat{\text{E}}$C = 70 - 40 = 30°.

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