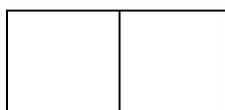


Slicing game

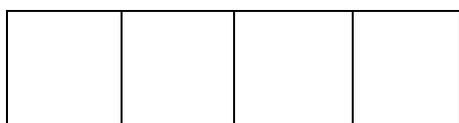
With scissors and as few cuts as possible, separate each of these strips of paper into single boxes.

(You can stack pieces on top of each other and cut them simultaneously. That counts as one cut).

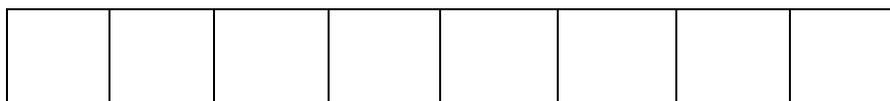
2 boxes



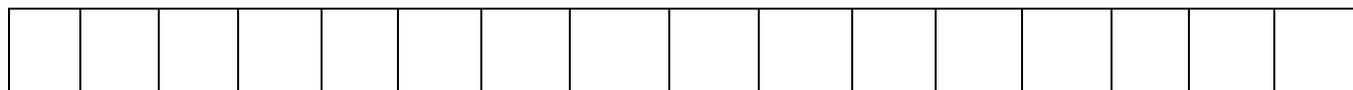
4 boxes



8 boxes



16 boxes.



How many cuts did you need each time?

Can you make a prediction for how many cuts you will need for a strip with:

32 boxes

64 boxes

128 boxes

Now try the same exercise with other strips:

3 boxes

5 boxes

7 boxes

9 boxes

14 boxes

18 boxes

35 boxes

Suppose you have a strip with a number of boxes which we call N. Can you use N to write a formula for the smallest number of cuts needed for this strip?

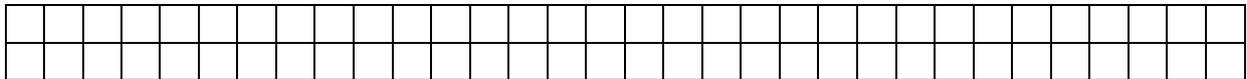
How many cuts will you need for a strip of:

N=100 boxes

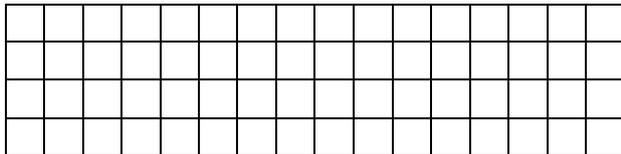
N=500 boxes

N=1000 boxes

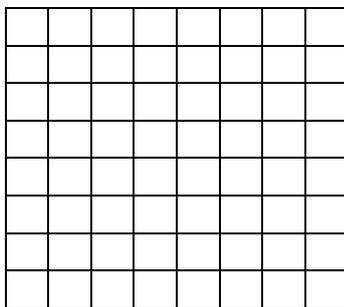
Which of the following can be cut into small boxes faster:



- a chocolate bar of 2 by 32 boxes



- a chocolate bar of 4 by 16 boxes



- a chocolate bar of 8 by 18 boxes

- or a chocolate bar of 64 boxes in a row?

How many cuts do you need for a grid of 2^x by 2^y boxes?

Guessing game

Pick any number you like between 1 and 100. I will try to guess your number, if I'm wrong please tell me if your number is larger or smaller than mine, and I will try another guess. What do you think will be the maximum number of guesses I need until I find your number?

Let's play....

Now you try

Can you explain your strategy?

How about if you pick any number you like between 1 and 1000?

Between 1 and N for some number N?

Answers

In the case of strips whose number of boxes is 2^n , the most efficient strategy is to cut them in the middle, then stack the new strips and cut them in the middle, etc for a total of n cuts.

In the case of strips whose number of boxes is N which is not of the form 2^n , repeat the same trick as above, cutting as close to the middles as possible. The answer is the natural number n such $2^{n-1} < N$, and N lies between 2^{n-1} and 2^n .

For the guessing game, the strategy is again to cut 100 in half, thus checking which half contains the mystery number, then check which quarter contains the mystery number, then which eighth, etc.

For example, if the mystery number is 100, the guesses could be:

50 – too small

75 – too small

88 – too small

94 – too small

97 – too small

99—too small

100 –good guess.

7 guesses may be necessary before finding the right answer

$$2^6 < 100 < 2^7.$$

(The initial intuition might be that a much larger of guesses would be necessary).