How do computers count numbers?

Imagine that you can only count with the digits 1 and 0, how would you be able to add, subtract or do anything else that you normally do with numbers whose digits go from 1 to 10?

A computer can only store a 0 or a 1 but it can store lots of them. E.g. the number 123 is stored as 1111011 on the computer. We call these **binary** numbers.

Activity 1

Look at the table below. Can you figure out what the next number in the Binary number column is? How about the rest?

Decimal number	Binary number
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	

Activity 2

We know how to add decimal numbers, e.g. 10+7=17, we can also add binary numbers e.g.

1+1=10 (this not ten!!)

If we look at our table we see 11 in binary corresponds to 3 in decimal.

You can add 11+11 like this:

(remember 1)

\+ 11

11

110

Try adding the following, some are already filled in.

Decimal number	Binary number
0+1=1	0+1=1
1+1=2	1+1=10
1+2=3	1+10=
1+3=4	1+11=
2+2=4	
	10+11=
	11+11=
	110+11=
	101+11=
	111+10=
	10+101=

Activity 3

Again we know how to subtract decimal numbers, e.g. 15-5=10, again we can do the same with binary numbers e.g.

11-1=1 (in binary)

Again try the following, as usual some are already filled in. Some subtractions in decimal haven't been converted to binary, can you help fill them in?

Decimal number	Binary number
2-1=1	10-1=1
3-1=2	11-1=10
3-2=1	11-10=1
	101-1=
	110-10=
	111-11=
	110-11=
	110-11=
	111-10=
8-3=	
9-4=	
10-2=	
10-3=	

Activity 4

When you were doing the last activities did you notice any patterns?

Is there anything special about the numbers 2,4,8,16 ?

Decimal number	Binary number
2	10
4	
8	
16	
32	
64	
128	
256	

Activity 5

Try adding the following numbers in binary: 1+2 + 4 + 8 + 16 +32.

What number is this?

What happens if you add 1 to this: 1+2 + 4 + 8 + 16 +32 + 1 = ... Try this in binary.

What can you say about the result?

Can you predict the results of the following operations:

1+2 + 4 + 8 + 16 +32 +64+ 1 =....

1+2 + 4 + 8 + 16 +32 +64+ 128+ 1 =....

Can you formulate a general rule?

Ternary numbers

Imagine now that you can only count with the digits 2, 1 and 0, how would you be able to add, subtract or do anything else that you normally do with numbers whose digits go from 1 to 10?

In this case we say we work with **ternary** numbers.

Activity 1

Look at the table below. Can you figure out what the next number in the Ternary number column is? How about the rest?

Decimal number	Ternary number
0	0
1	1
2	2
3	10
4	11
5	12
6	20
7	21
8	22
9	100
10	101
11	102
12	110
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	

Activity 2

We know how to add decimal numbers, we can also add ternary numbers e.g.

11+20=101 (in ternary)

How would the operation above look in decimal numbers?

Try adding the following, some are already filled in.

Decimal number	Ternary number
0+1=1	0+1=1
1+1=2	1+1=2
1+2=3	1+2=10
1+3=4	1+10=11
2+2=4	2+2=11
	10+11=
	111+12=
	102+21=
	111+10=
	10+101=
3+3=	
1+6=	
1+3+9=	
9+9=	
1+3+18=	
1+6+9=	
1+6+18=	
1+3+9+27=	
27+27=	
1+3+9+54=	
2+6+18+54=	
81=	

Can you write the following numbers in ternary:

30	
37	
60	
64	

Activity 4

When you were doing the last activities did you notice any patterns?

Decimal number	Ternary number
3	10
9	
27	
81	
273	
	1000000

Activity 5

Try adding the following numbers in ternary: 2+6 + 18 + 54 + 162+546.

What happens if you add 1 to the number above?

Did you notice a certain pattern?

Could you find a similar pattern when trying to write all numbers using only the digits 1,2,3,4?

Can you find a similar rule for other numbers?

Can you prove it?