Sets

Introduction: This lesson is an introductory lesson to the topic of sets. The students will be introduced to the ideas of sets, elements, union and intersection in an indirect way.

- ***A set is a clearly defined collection of objects.***
- ***An element is the name given to an object within a set.***
- ***The intersection of two sets A and B is the list of elements common to both sets.***
- ***The union of two sets A and B is the list of all the elements in both sets.***

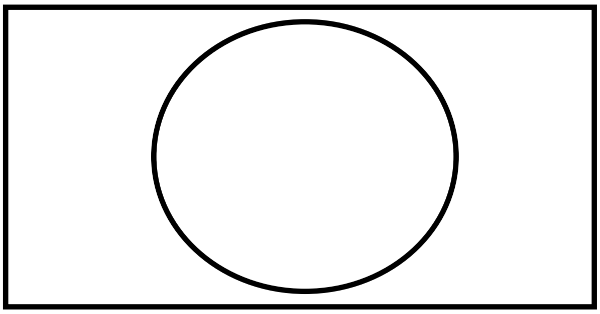
Activity 1: Class introduction to the notion of sets

Resources:

- 3 hula hoops (if you don't have access to these then you can make circles out of string, with each being roughly the same size).
- Each student must have a piece of paper with their name written on it.
- Large table or group of 3/4 classroom tables grouped together on which you can lay the hula hoops in different patterns.

Tutor instructions:

1. Begin by placing a single hula hoop on the desk like this:



Have the circle represent different sets relating to the lives of the students. For example:

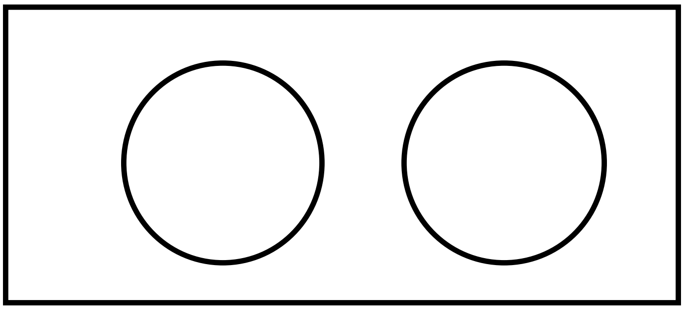
- The set of students who play a sport
- The set of students who play an instrument
- The set of students with blue eyes
- The set of students with brown hair

If a student belongs to the set then they place their name inside the hoop. If a student does not belong to the set they leave their name on the table outside the hoop.

> The table represents what is known as the universal set in this task. The universal set is the collection of all objects, in the example above the universal set corresponds to the collection of all student names. Through this exercise we are exposing the students to the notion of the universal set without confusing them with the formal terminology.

Introduce the term 'element' in this step. Introduce the term in a concrete way so that the students can relate to it. For example: take the set of students who play GAA in the class and then of those explain that "Mary" is an **element** of that set. As the lesson progresses you should ask the students 'Who is an element of this set?' every time you create a new set to familiarise them with the term.

2. Next, place 2 hoops on the table like this:



Again, use each hoop to represent a set. For example: One hoop could represent the students in the class with blonde hair and the other could represent the students with black hair. Again, the students should place their name in the hoops if they belong to either of these sets and should place it outside the hoops if not.

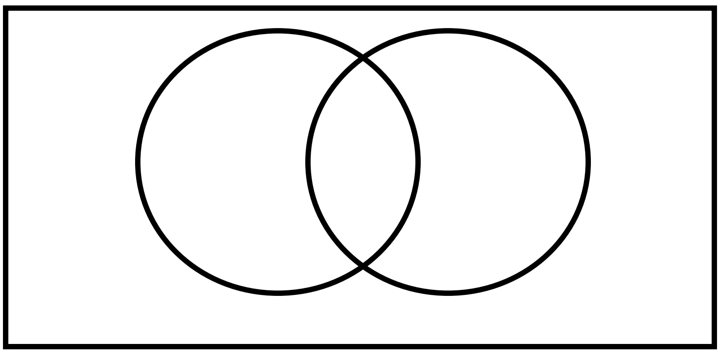
Progress then from this to two sets in which the students could be an element of both sets simultaneously. For example: one hoop could represent the students in the class with brown hair and the other could represent the students in the class with brown eyes.

This part of the exercise is designed to inspire discussion among the students. They should question what they do if they belong to both of the sets represented by the hoops.

3. When students have identified the problem of being an element of 2 sets simultaneously ask them:

Do you think there's a way we could arrange the hoops so that the students who have brown hair and brown eyes could be in both hoops at the same time?

Allow the students to make suggestions and try each arrangement suggested until they come to this arrangement:

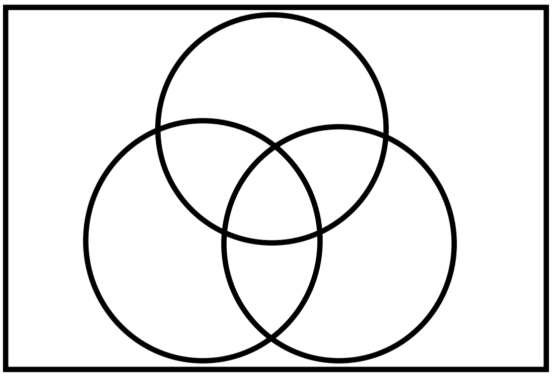


Try a number of combinations of set titles which may cause an intersection.

(Combinations of eye and hair colour usually work well for this)

Introduce the term ‘intersection’ at this point of the lesson explaining that the part where the circles overlap is given this name. As the lesson progresses you should ask the students ’Who is an element of the intersection of these two sets?’ to familiarise them with the term.

4. Next ask the students what would happen if someone belonged to 3 sets at the same time. Where would we place the third hoop? Again, try the different arrangements until they arrive at:



Again, use the hoops to represent sets and ask the students to place their names in the appropriate area. Try a number of combinations of set titles which may cause intersections. For example: one hoop could represent the students with blonde hair, another could represent the students with blue eyes and the other could represent the students who play a sport.

Activity 2: Speedy Sets

Resources:

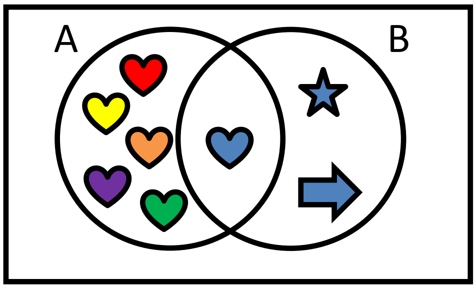
- Colour copies of template 1 for each team which need to be cut out to make counters (these may be easier to use if copied onto card).
- A copy of Template 2 for every team.
- A copy of the question tiles from Template 3 for every table (these should preferably be copied onto card for re-use).

Tutor instructions:

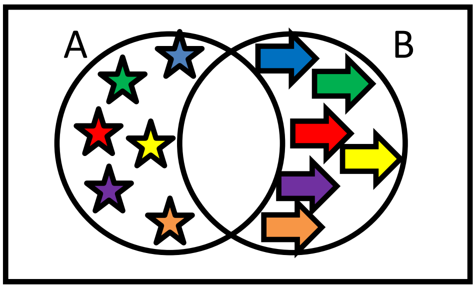
- This is a competitive task where 2 teams are racing one another.
- Split the class into teams of 3 or 4.
- Set up the classroom so there are 2 teams at every group table. (If you have the time you can set up a little league on the board where winners play winners etc.)
- Give each team the 3 sets of counters from Template 1.
- Give each team a copy of the Venn diagram template (Template 2).
- Place a set of question tiles on each table (instruction side faced down).
- **Explain the Rules Clearly.** Give each student a copy of the student instructions on which these rules are printed.

Solutions:

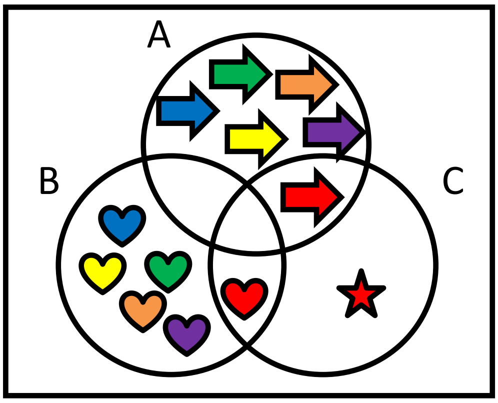
1. A={The set of heart counters},
   B={The set of blue counters}



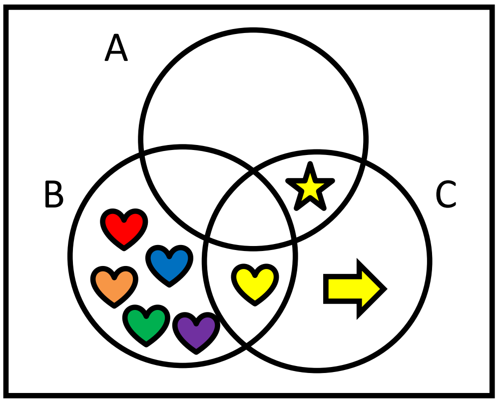
2. A={The set of star counters},
   B={The set of arrow counters}



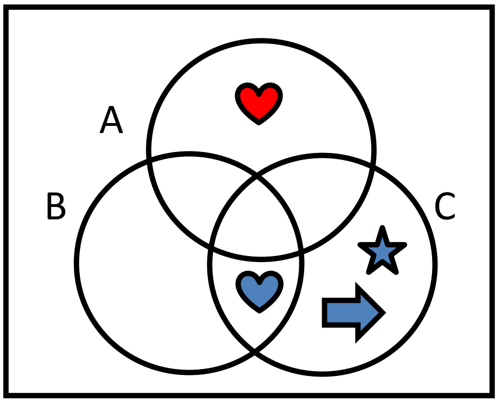
3. A={The set of arrow counters}
   B={The set of heart counters}
   C={The set of red counters}



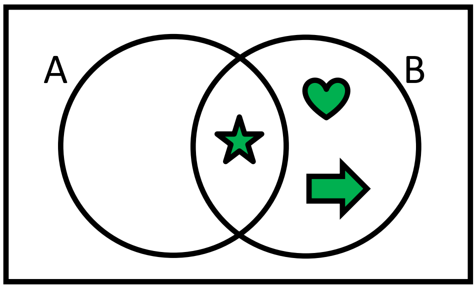
4. A={The set of yellow star counters}
   B={The set of heart counters}
   C={The set of yellow counters}



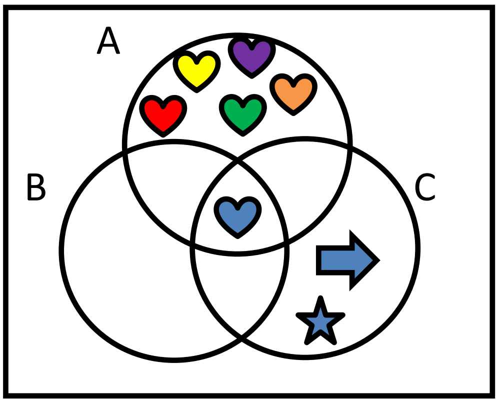
5. A={The set of red heart counters}
   B={The set of blue heart counters}
   C={The set of blue counters}



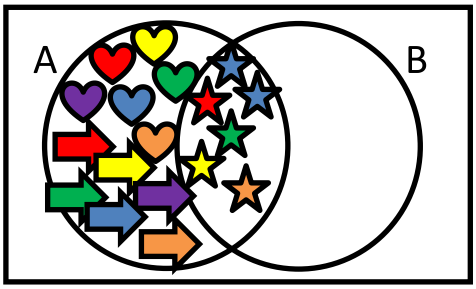
6. A={The set of green star counters}
   B={The set of green counters}



7. A={The set of heart counters}
   B={The set of blue heart counters}
   C={The set of blue counters}



8. A={The set of all counters}
   B={The set of star counters}



Student Instructions:

Each team has 3 sets of multi-coloured counters sorted by shape. You must sort the counters in response to the sets that are written on the back of the question tiles using the correct circle diagram. You must try to place the counters in the correct area of the circles.

Each team can come up with their own buzzer noise. You must try to answer the question by sorting the counters correctly and faster than the other team. As soon as you think you've done it correctly make your buzzer noise! The first team to answer the question correctly gets a point. If you make your buzzer noise but you get the puzzle wrong then the point goes to the other team.

Rules:

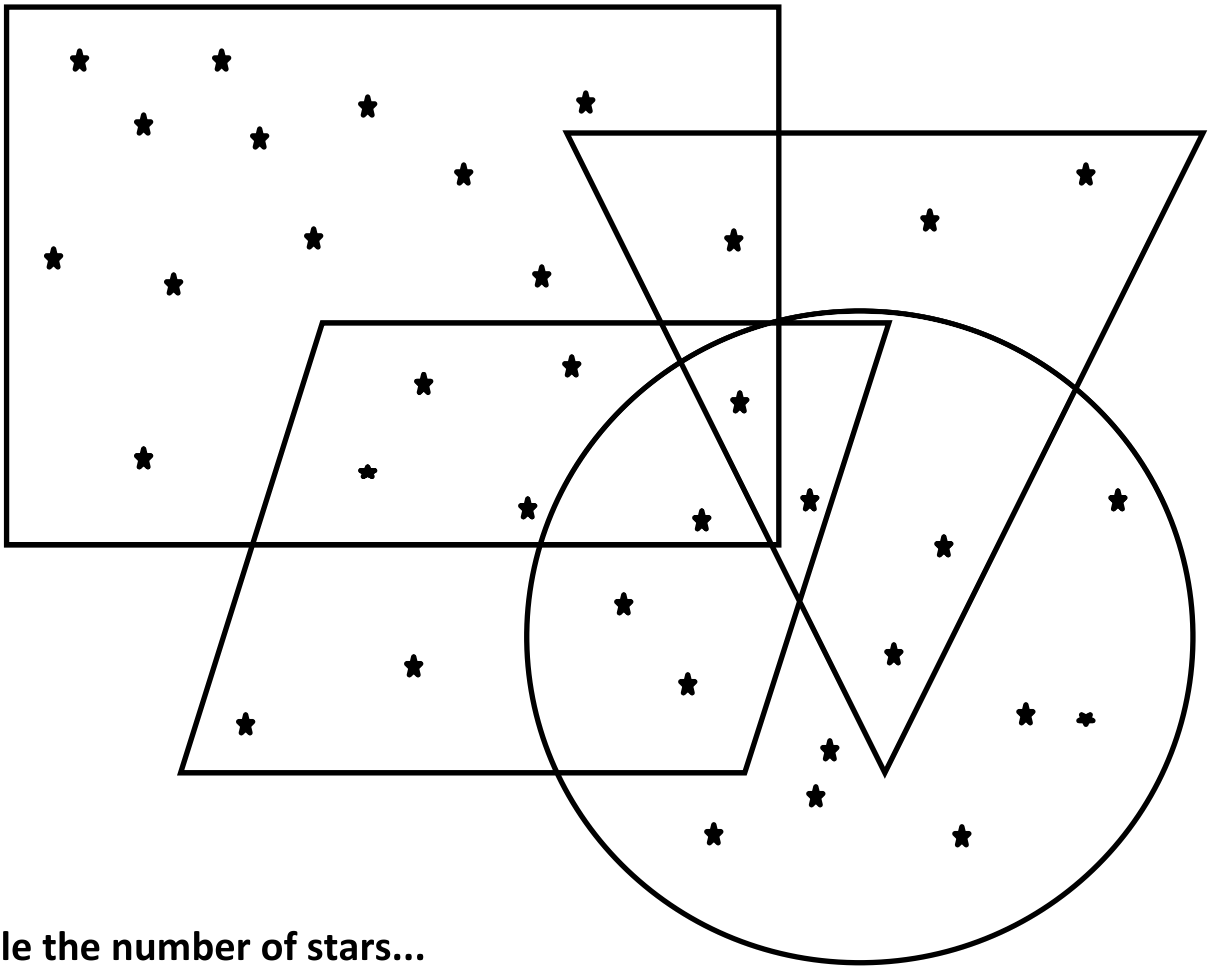
- Each team must select one team member to be "sorter" (You must take turns). When someone is selected as sorter they are the only person who can touch the counters. Other team members can talk to them but they must **not** touch the counters.
- Only when both teams are ready can a tile be turned over. The tile must be turned over by a student who is **not** one of the nominated sorters.
- When a sorter thinks they have completed the challenge correctly they must make the team's buzzer noise. If they are correct and were the first team finished then they receive a point. If they are incorrect the other team receives a point.

Good Luck!

Take Home Problem:

***1 copy per student**

STAR TRACKER



Circle the number of stars...

a) In the ◯ but not in the ▽ , ▱ or ▭ — 6 8 7

b) In the ▽ but not in the ◯, ▱ or ▭ — 3 5 2

c) In the ▱ but not in the ▭ , ◯ or ▽ — 2 3 11

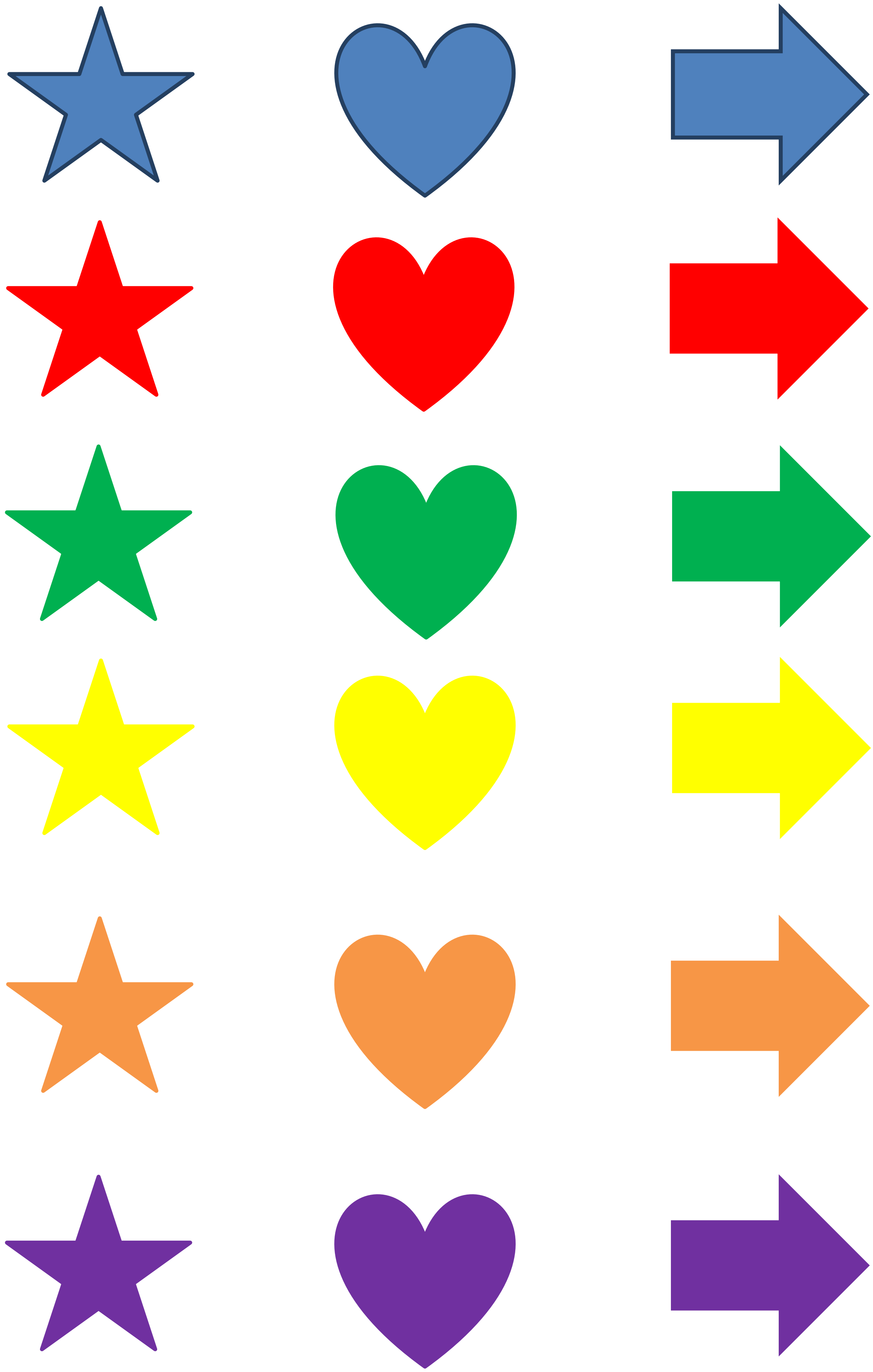
d) Common to the ◯ and ▽ but not to the ▱ or ▭ — 12 2 4

e) Common to the ▭ and ▱ but not to the ▽ or ◯ — 4 6 10

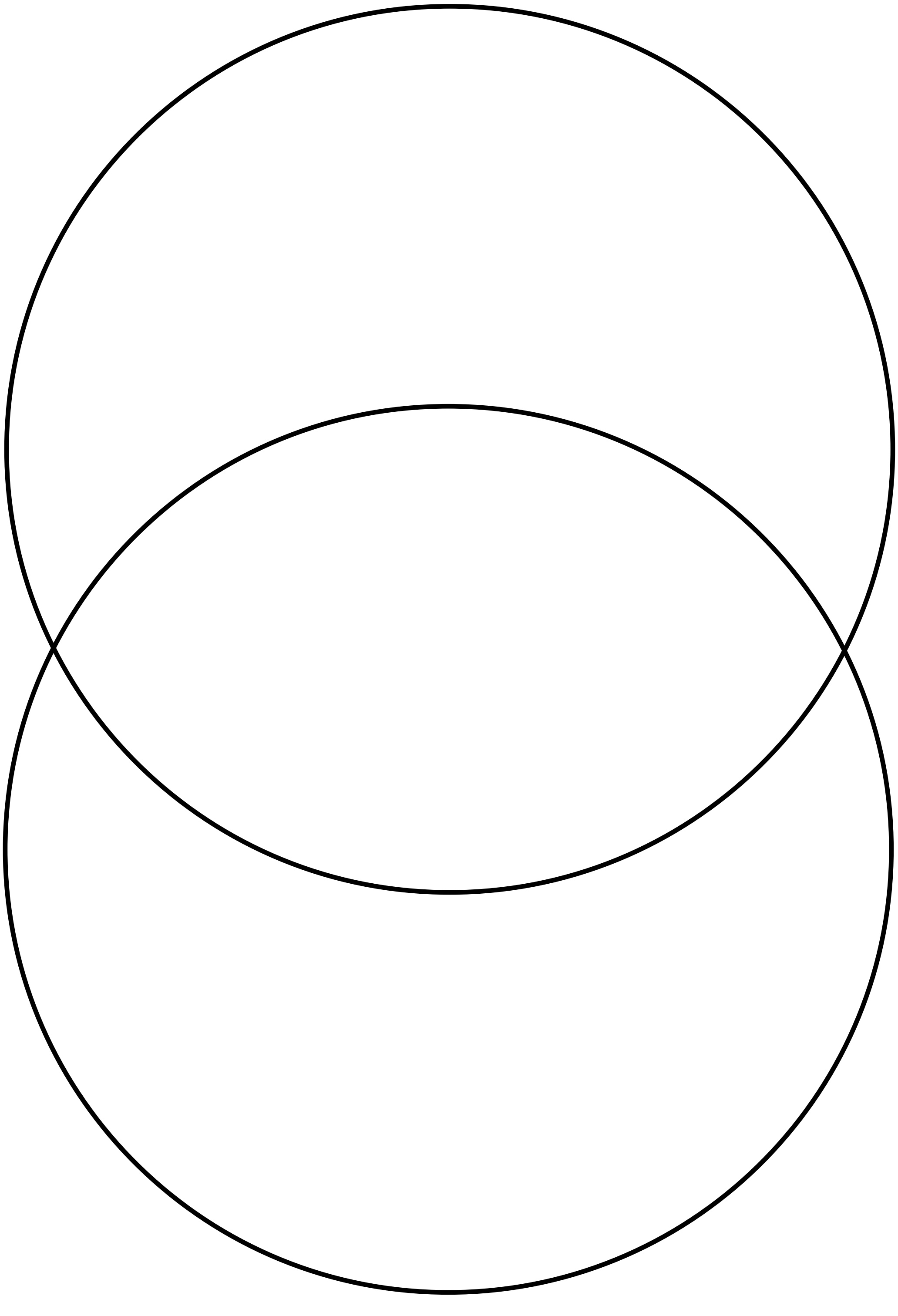
f) Common to all four figures — 1 2 9

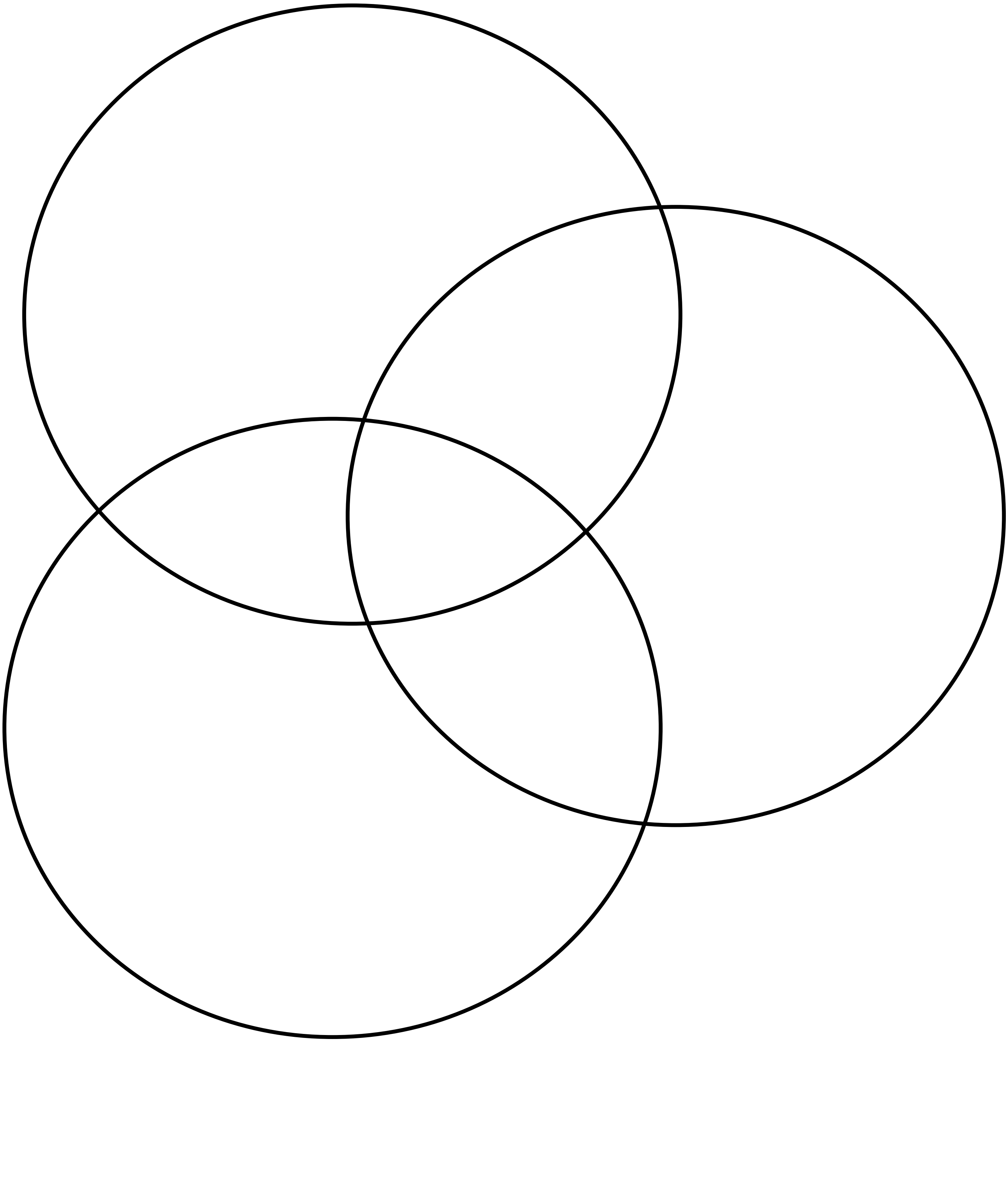
Solutions: a)7, b) 2, c)2, d) 2, e) 4, f)1

Template 1



Template 2





Template 3

1.

A={The set of heart counters},

B={The set of blue counters}

2.

A={The set of star counters},

B={The set of arrow counters}

3.

A={The set of arrow counters}

B={The set of heart counters}

C={The set of red counters}

4.

A={The set of yellow star counters}

B={The set of heart counters}

C={The set of yellow counters}

5.

A={The set of red heart counters}

B={The set of blue heart counters}

C={The set of blue counters}

6.

A={The set of green starts counters}

B={The set of green counters}

7.

A={The set of heart counters}

B={The set of blue heart counters}

C={The set of blue counters}

8.

A={The set of all counters}

B={The set of star counters}

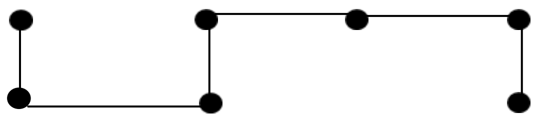
Hamiltonian Circuits and Paths

Introduction: This lesson is an introductory lesson to the topic of Hamiltonian Circuits and Paths. The students will learn the difference between a path and a circuit and also the characteristics of Hamiltonian paths and circuits.

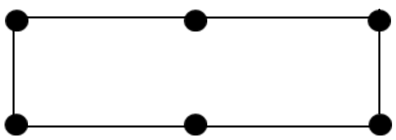
Background information for tutor:

- **A Hamiltonian path is a graph that visits each vertex exactly once.**
- **A Hamiltonian circuit is a graph cycle that visits each vertex exactly once except the start/end point which is visited twice in order to close the circuit.**
- **A circuit comes back to its starting point while a path does not.**

Example of a Hamiltonian path:



Example of a Hamiltonian circuit:



Activity 1: Museum circuits

Resources:

- A copy of Template 1 per student
- A pencil per student
- A rubber/eraser per student

Tutor instructions:

- This is an individual task
- Give each student a copy of the pages from Template 1
- Give each student a copy of the Student Instructions
- Ensure that each student has a pencil and rubber. Instruct the students to use these instead of pen so they can try out different paths.

Student Instructions:

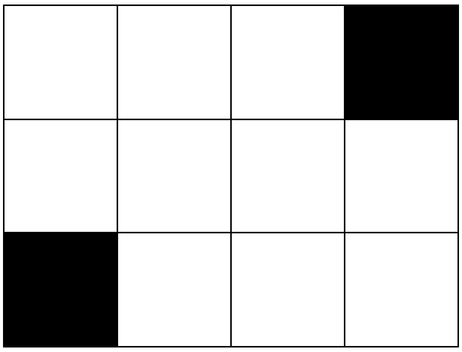
You are a security guard in a museum. The diagrams below represent a plan of the different floors of the museum. Each room is connected with all its neighbouring ones, horizontally or vertically but <u>not</u> diagonally. As the guard, you must visit every room on each floor once, except the darkened rooms which are closed for renovation, before closing the building. Your path must:

a) Go from a room to a neighbouring room

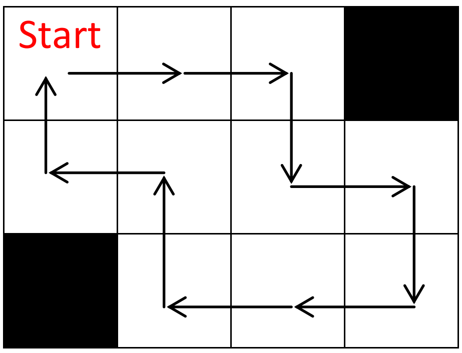
b) Pass through all the rooms (except the darker ones) once each

c) Finish back where you started.

Here is an example:



Solution:



Activity 2: Speedy Circuits

Resources:

- A copy of each page in Template 2 per student (not to be given to them individually)
- A pencil per student
- A rubber/eraser per student

Tutor Instructions:

- This is a competitive task where students are racing one another to complete circuits.
- Set up the classroom such that there is a desk per every pair of students.
- Place 2 piles of the copies of each page of Template 2 on each desk face down in the same order so that when the students turn them over they will both have the same diagram.
- Place 2 pencils and 2 rubbers on each desk.
- Split the students into pairs.
- Give each student a copy of the student instructions and read through the rules with the students before beginning the activity.
- The game continues until every student has completed all 5 challenges.

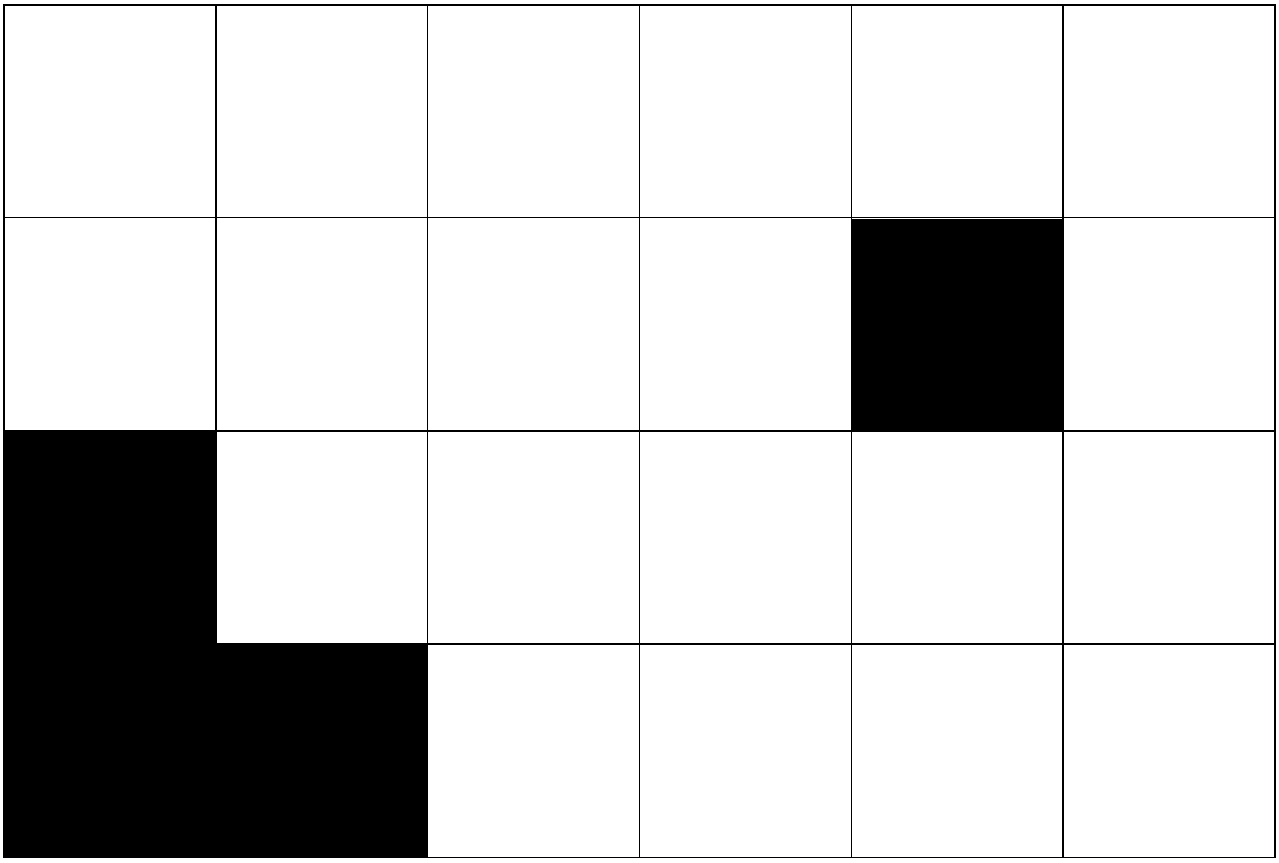
Student Instructions:

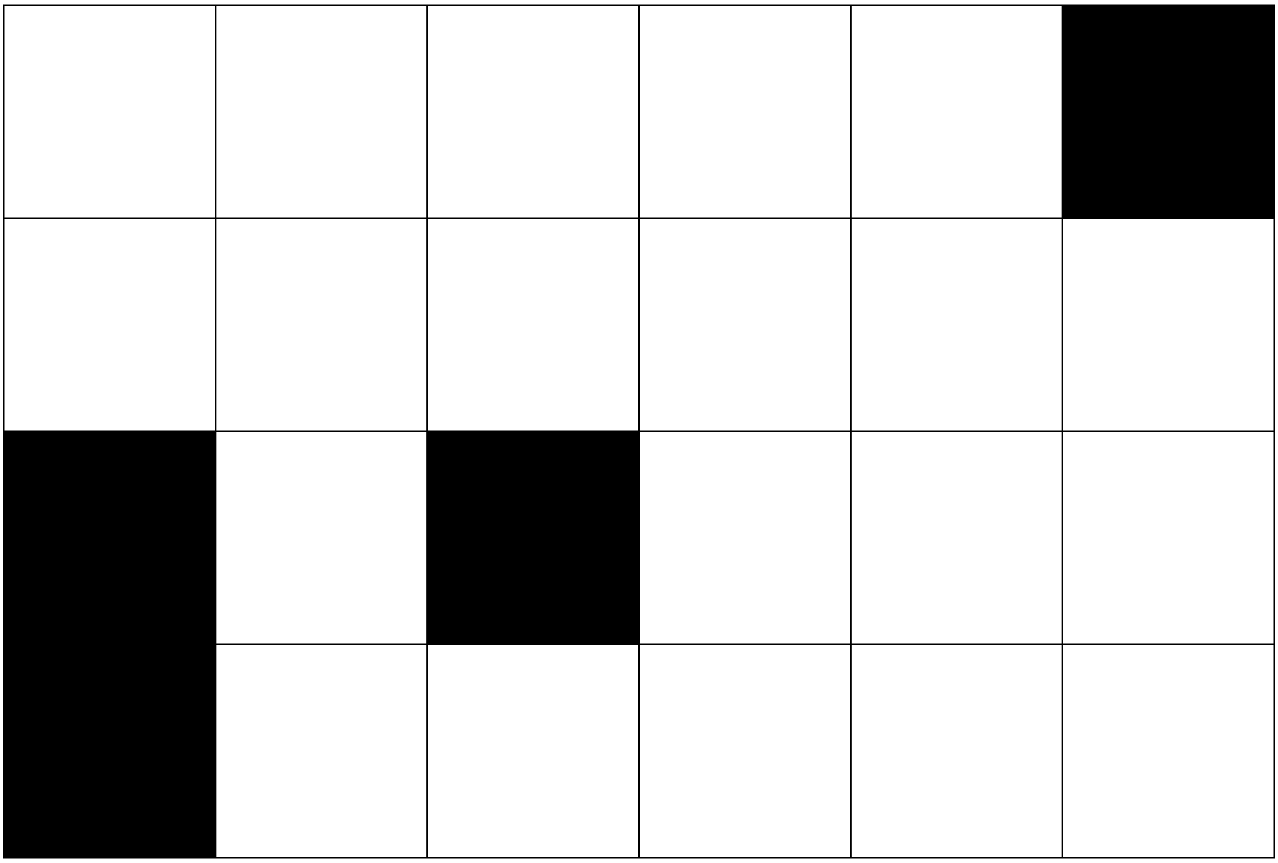
In this activity you are racing your classmates to try and complete a path first. When you are instructed to, you will turn over the page that is in front of you on the desk. The page will have a diagram on it like the diagrams of the museum in the first activity. You must race your opponent to try and be the first person to find the path around the diagram. Here are the rules:

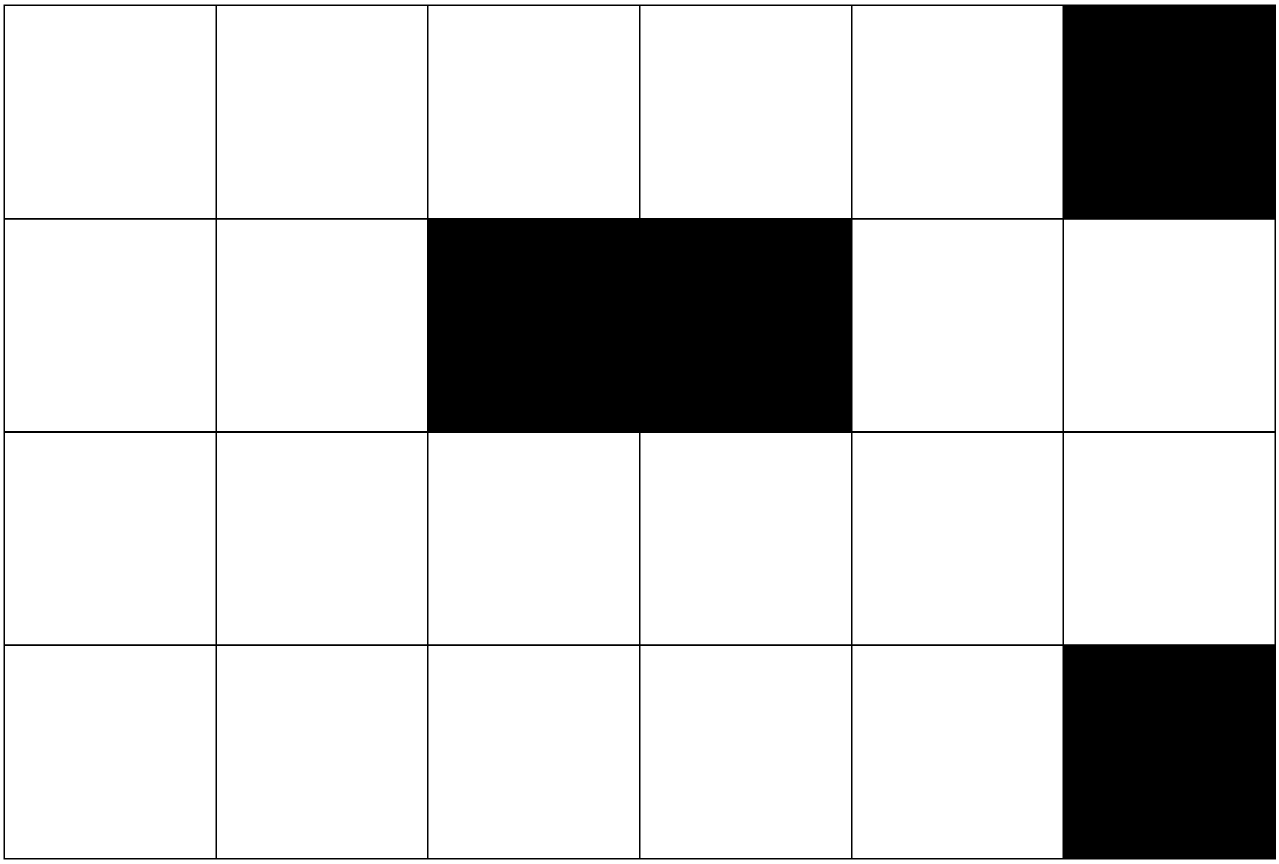
1. You may **not** turn over your page until you are told to do so
2. The path must go through every box on the diagram just like in the first activity
3. You must complete the path using pencil
4. You may rub out and try again as many times as you like
5. You must drop your pencil if you or your opponent says they are finished
6. The winner of the race must move onto the next desk and the person who lost stays at the desk they are at
7. The game continues until everyone has played using all 5 diagrams

Good Luck!!!

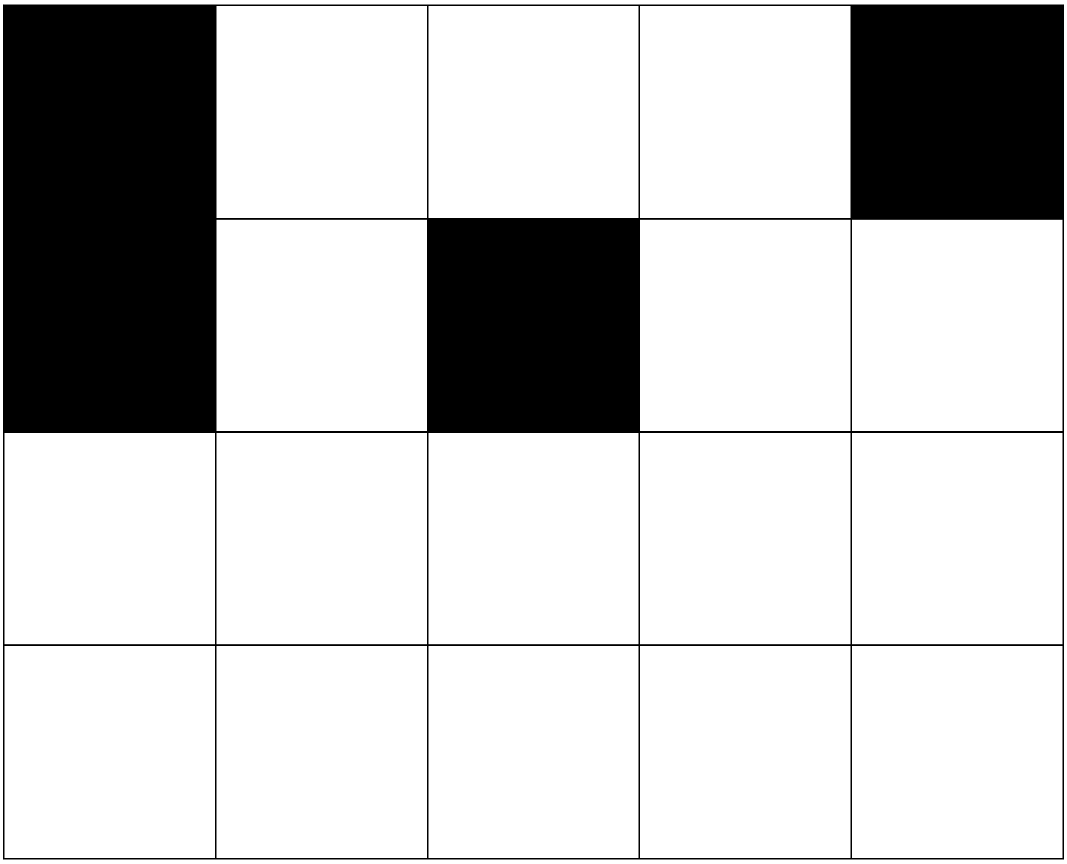
Template 1

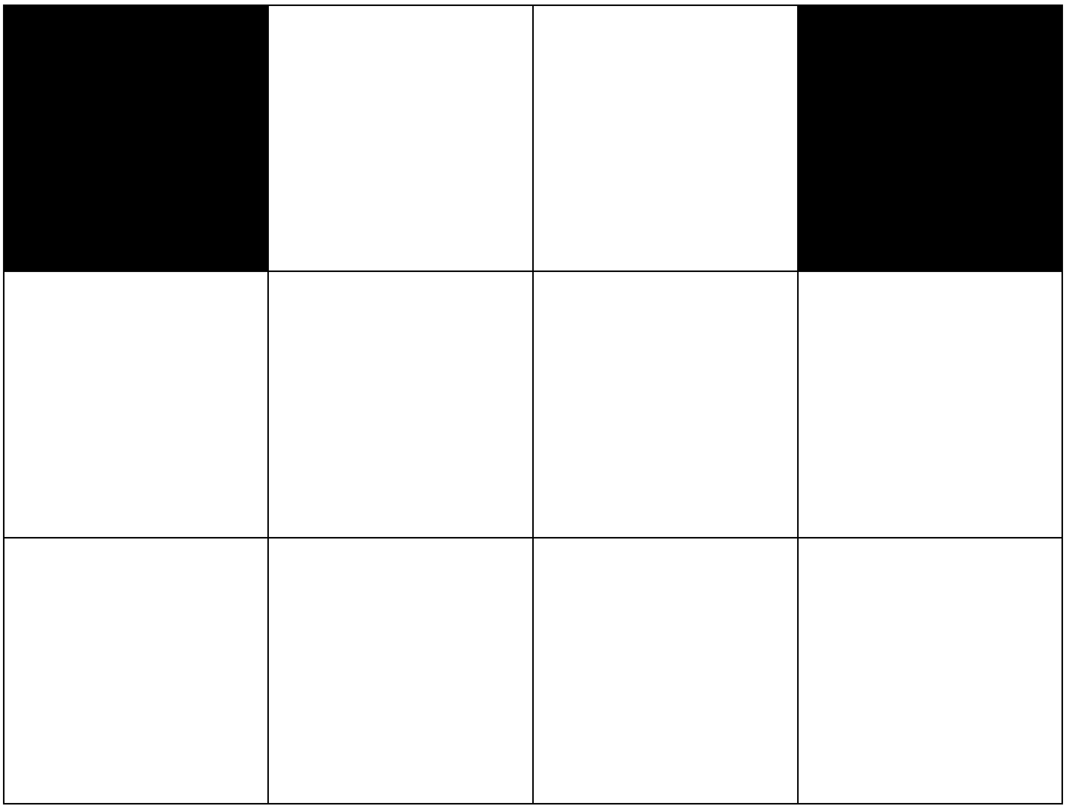


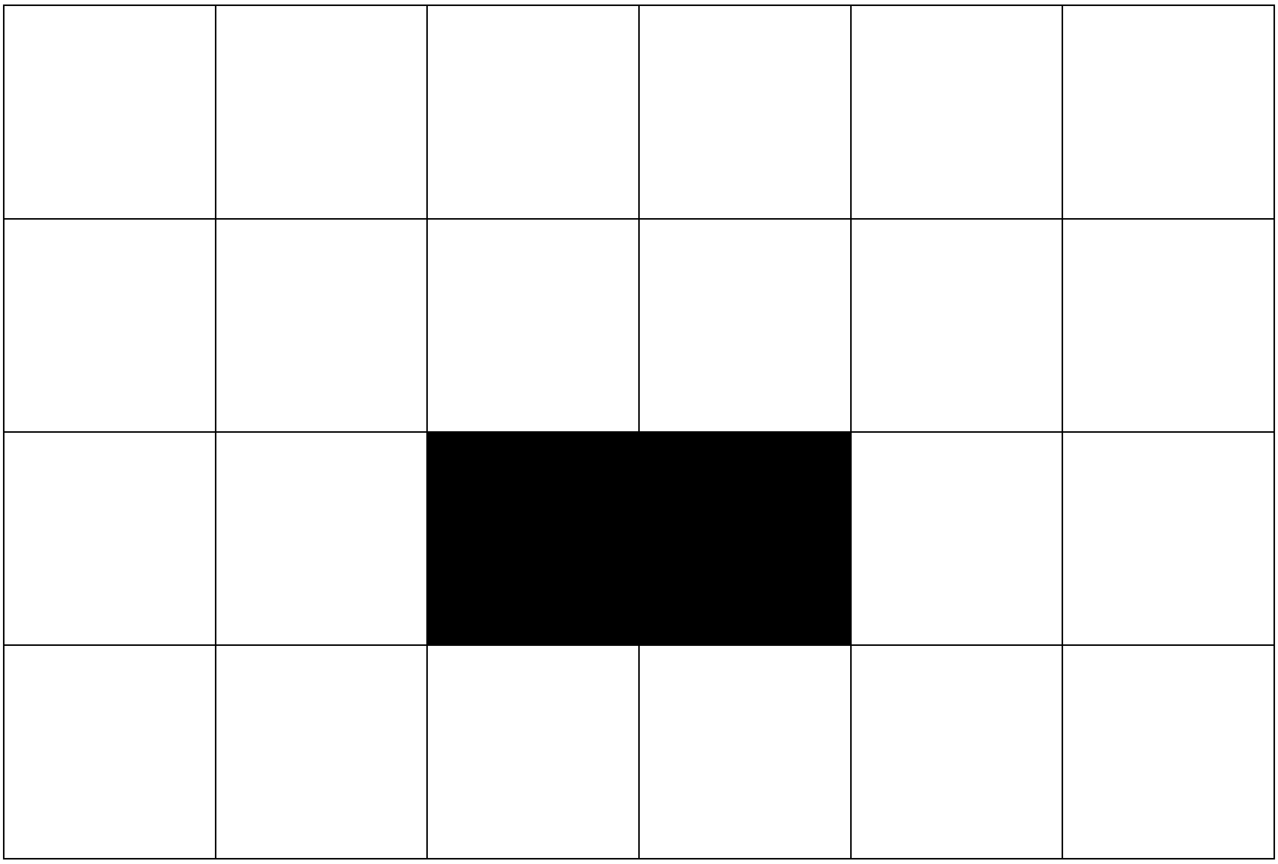


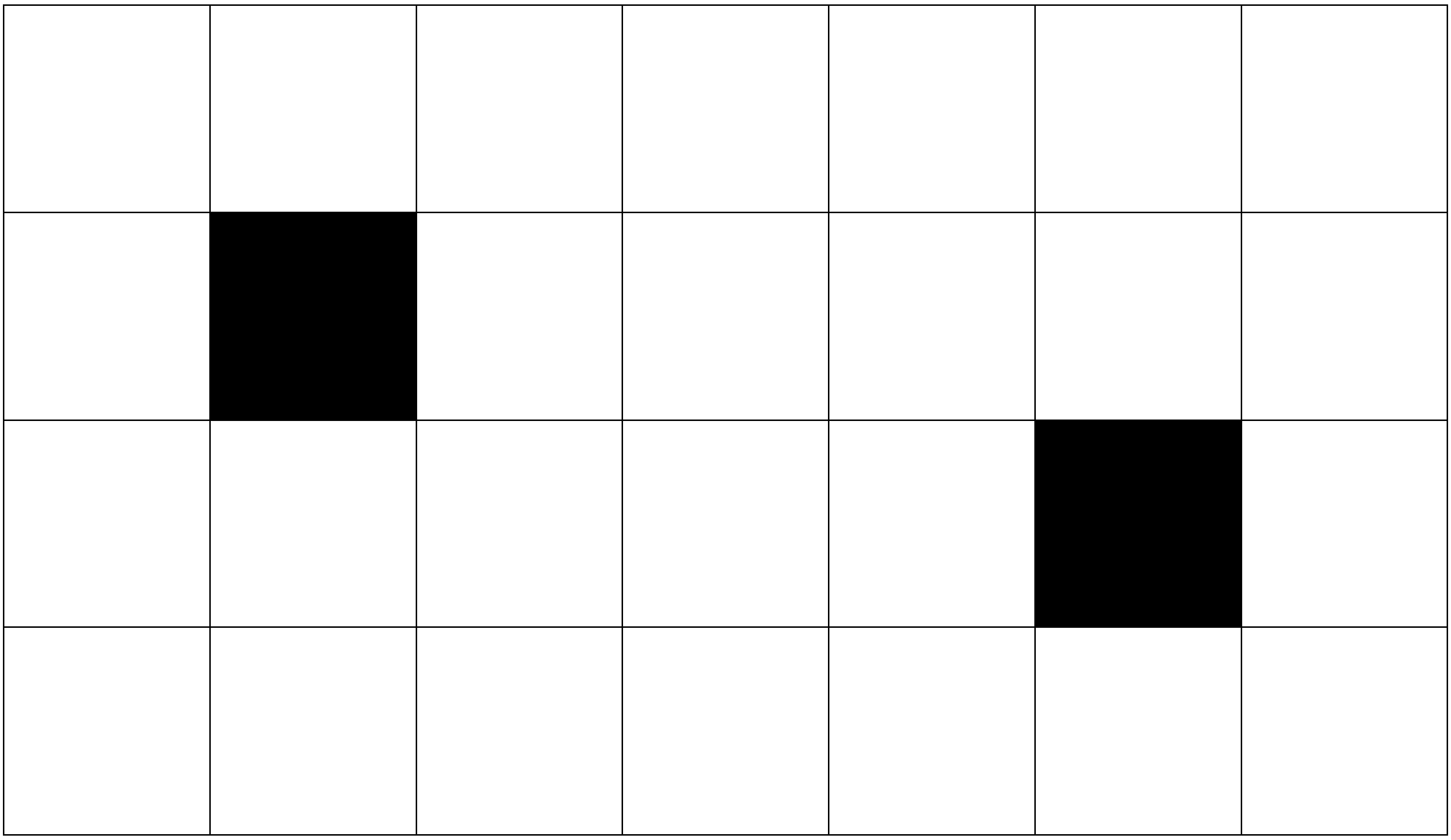


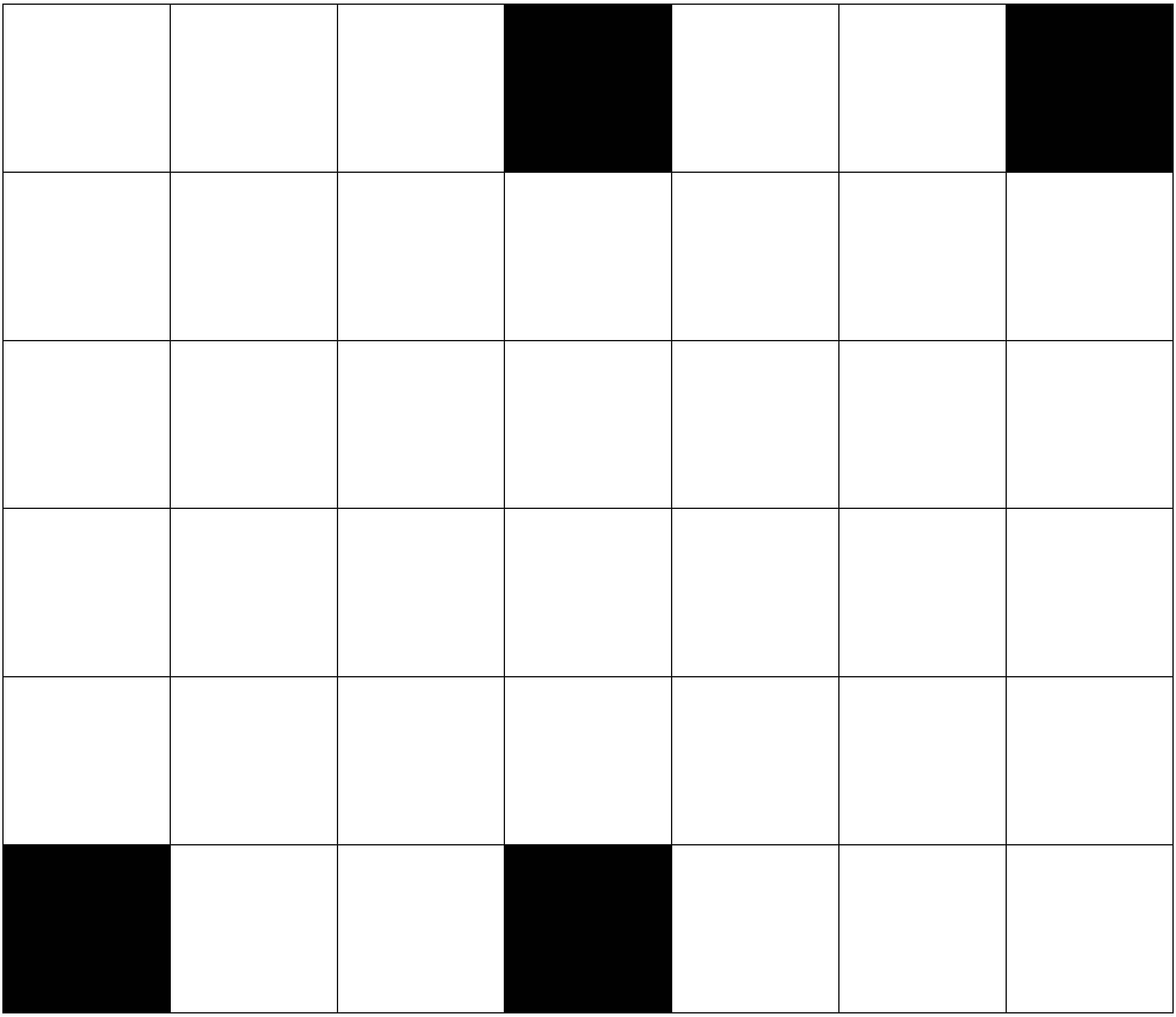
Template 2





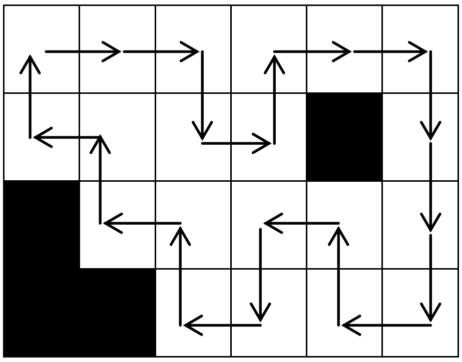


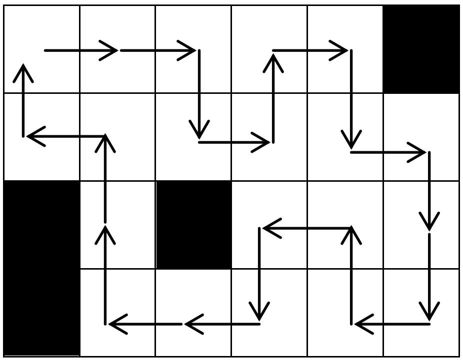


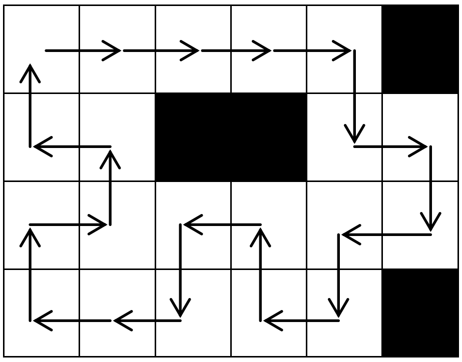


Solutions:

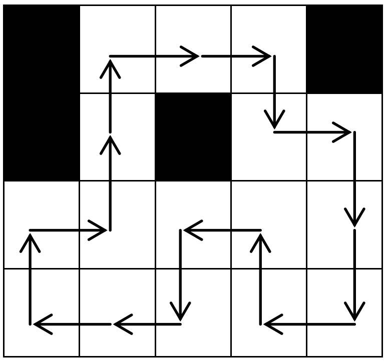
Activity 1:

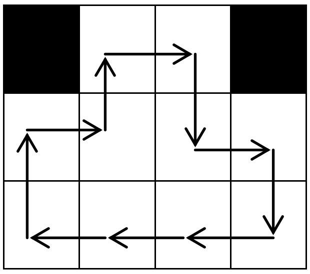


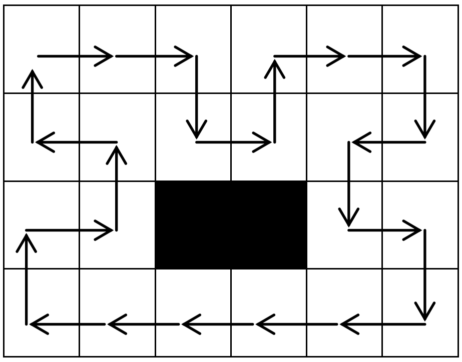


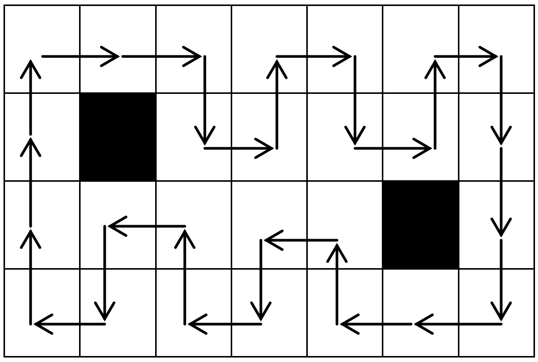


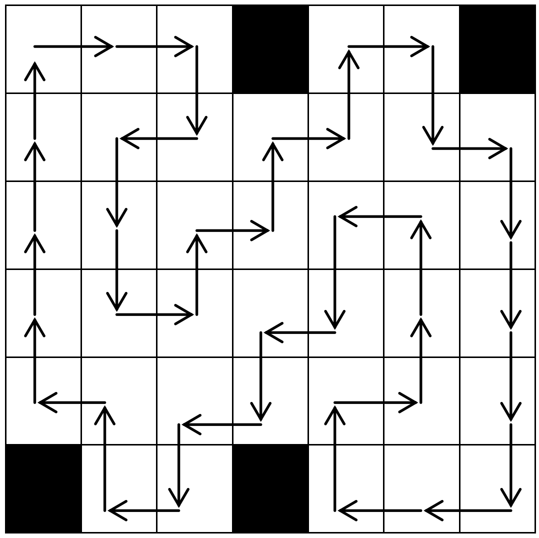
Activity 2:











Tessellations 1

Introduction: This is a geometry lesson. Students will be working on translations, rotations and symmetries in an indirect way through an Escher inspired artwork lesson.

Examples of Escher's Work:

Background Information for Tutor:

- A tessellation is a repeating pattern that tiles the plane without leaving gaps.
  - Perhaps relate this idea to things that the students can identify with, for example, bricks in a wall, tiles in their bathroom or kitchen etc.

You could use this point in the lesson to inspire some initial discussion among students by asking: 'Do you think we could tile a floor with any shape or would only some shapes work?'

Activity 1: Tiling the Wall

Resources:

- Copy of template 1 for each group (These shapes need to be cut out and may be easier to use if printed onto card).
- Some blank paper or the students can use A4 pads.

Tutor instructions:

- This is a group-work activity.
- Split the class into groups of about 4 (depending on your numbers).
- Give each group a copy of the shapes from Template 1.
- Ask the students to try and 'tile the wall' using each shape i.e. place the shape on the blank page, trace around it and repeat, trying to fill the page with no gaps.
- When they realise some of the shapes won't work ask them to try and combine some of the shapes to fill the page with no gaps. The last 3 shapes of Template 1 are the right sizes for these combination tessellations.
- Ask the groups to try and come up with some reasons as to why some shapes tessellate and others do not and why some combinations of shapes tessellate.

This last point is designed to inspire discussion among the students but it may be a little difficult for them to come to the right conclusion. In order to help this process you could perhaps ask the following prompting questions:

How many degrees are there in the angles of an equilateral triangle?

How many degrees are there in the corners of a square?

How many degrees are there in the angles of a pentagon/hexagon/octagon?

How many of the angles in a _______ would I need to make up 360 degrees?

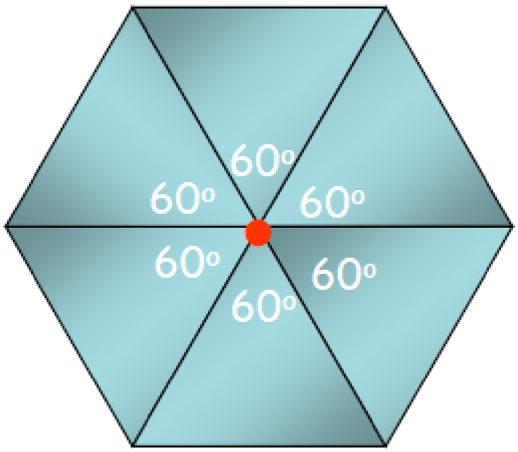
Student Instructions:

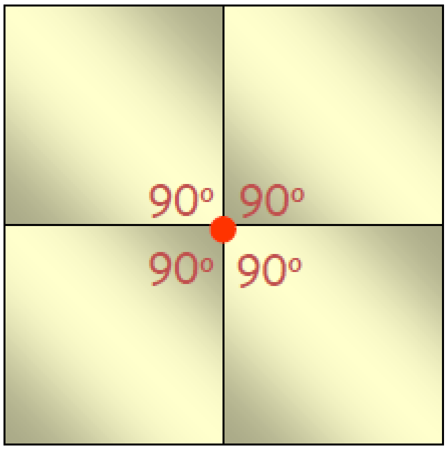
You are a group of builders and you have been asked to tile a bathroom wall using a tile in the shape of the cut-outs you have been given. You must check if it is possible to tile the wall for each shape you have been given.

1. Place your cut-out in the centre of a blank sheet of paper and trace around it.
2. Rotate or move your cut-out to repeat the shape joined onto the one you just drew.
3. Can you fill the whole wall with no gaps?
4. Can you combine different shapes to tile the wall? Use the smaller versions of the square and triangle to try combining shapes.
5. With your group try and come up with some reasons why only some shapes will tile the wall by themselves and why some combinations of shapes can be used to tile the wall.

Solution:

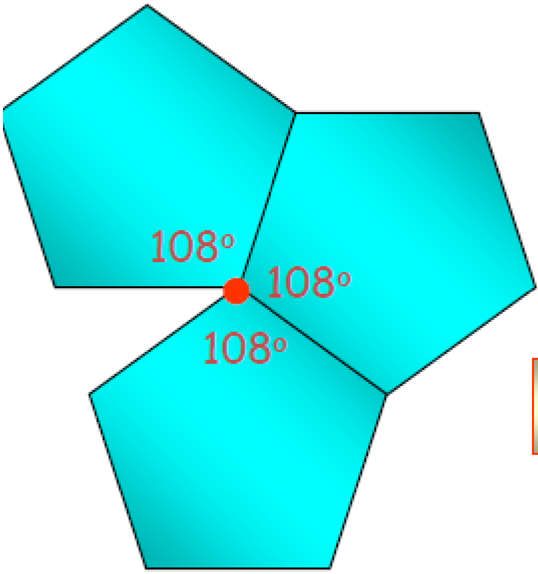
The sum of the interior angles about the indicated point must be 360



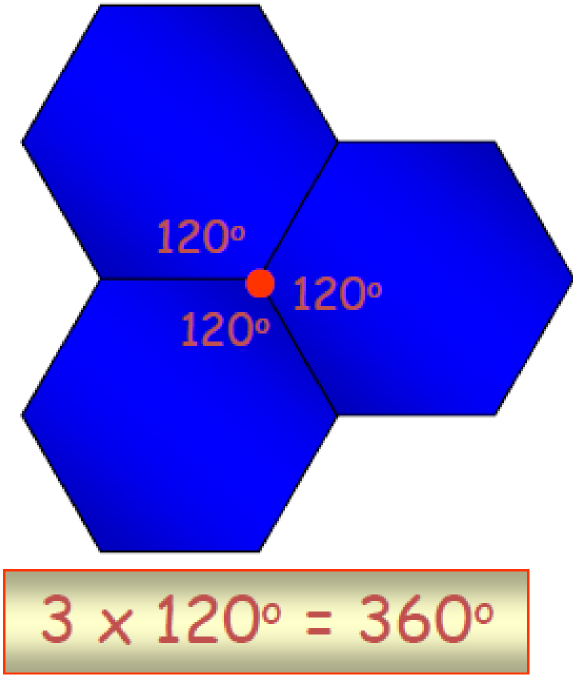


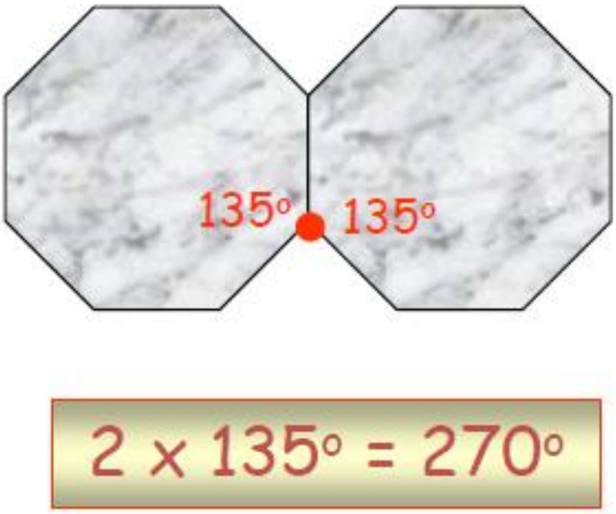
$6 \times 60^\circ = 360^\circ$

$4 \times 90^\circ = 360^\circ$

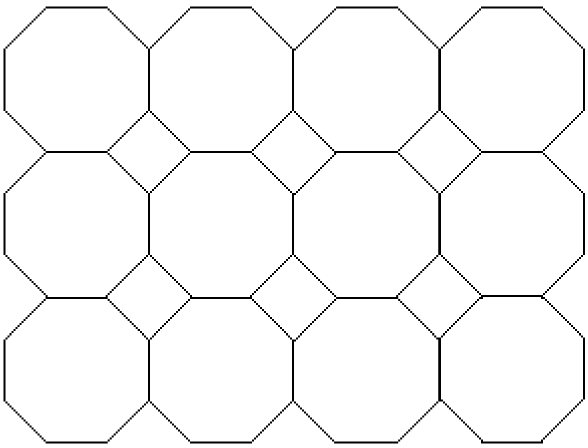


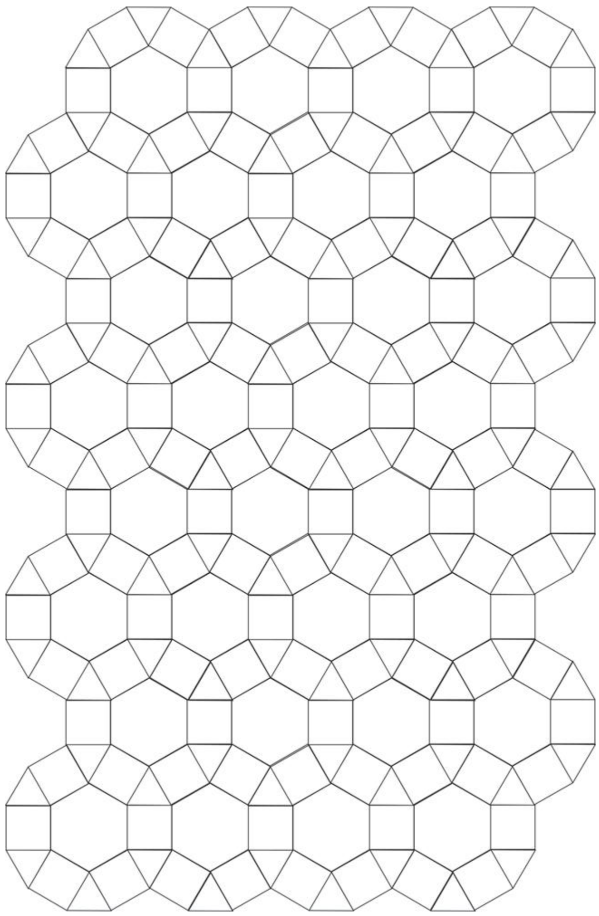
$3 \times 108^\circ = 324^\circ$



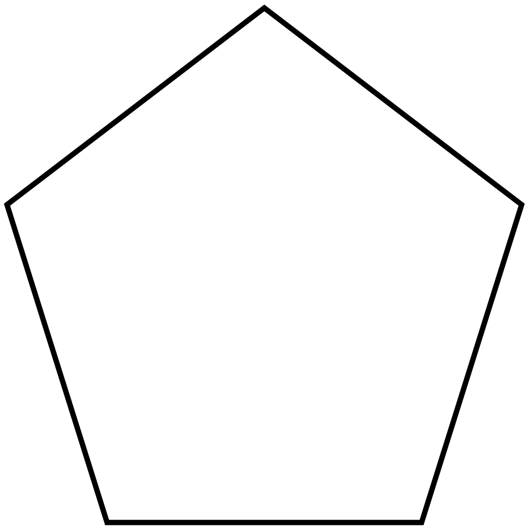


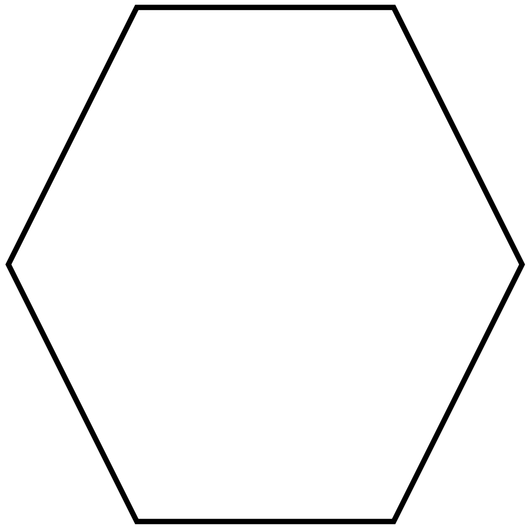
Examples of how you can combine shapes:

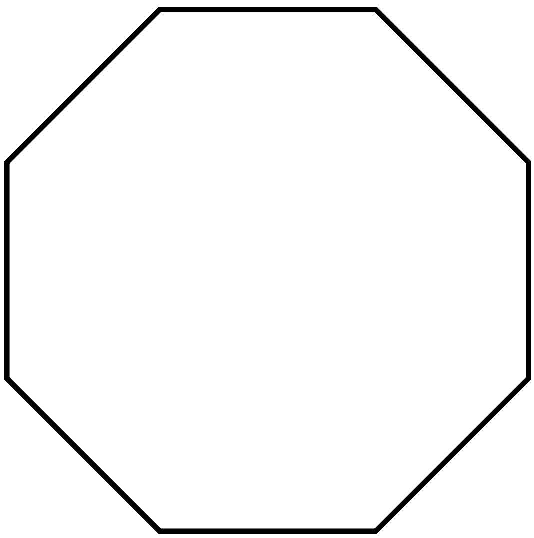


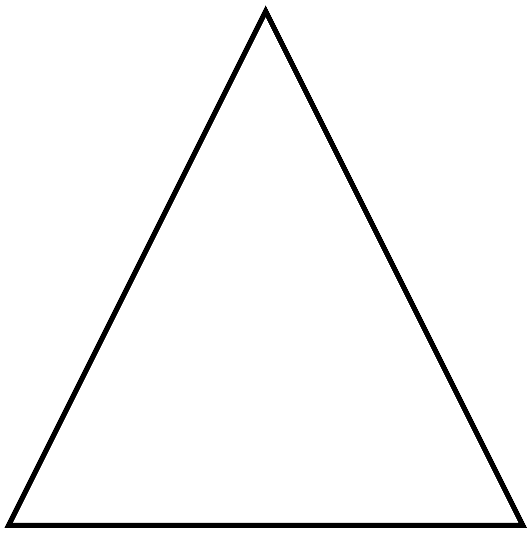


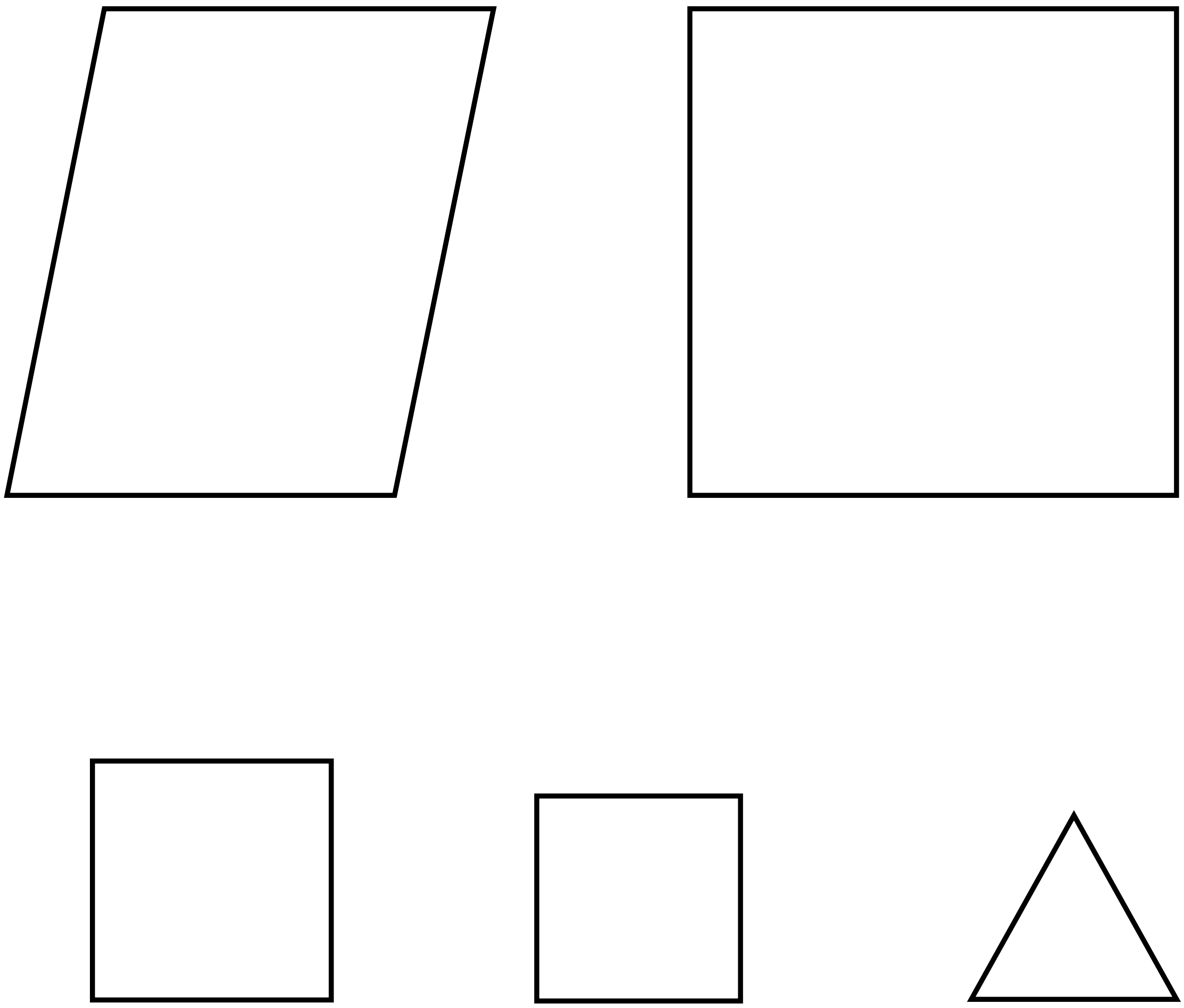
Template 1











Activity 2: Make Your Own Tessellation

(Depending on the length of your lesson this may be used as a take-home problem or could be started in class and finished at home)

Background Information:

- A translation is moving a shape without rotating or flipping it but by sliding it across the page. The shape still looks exactly the same; it is just located in a different place on the page.

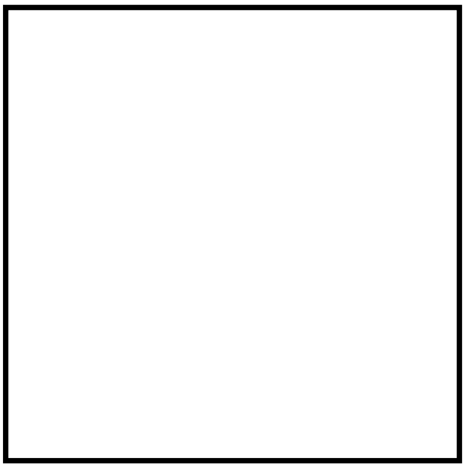
Resources:

- Copy of the 6cm x 6cm square in Template 2 per student (These should be printed onto card if possible)
- Scissors such that every student has access to one (Perhaps one to every 2 or 3 pupils)
- Enough sellotape such that every student has access to some (Perhaps one roll to every 2 or 3 pupils)
- 3 sheets of blank paper per student
- Colouring pencils

Tutor Instructions:

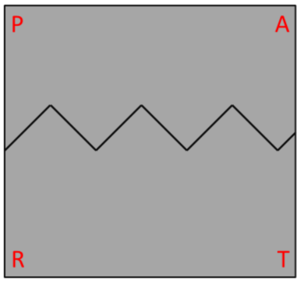
- This is an individual activity.
- The students will be translating their shapes in order to fill the page.
- Set up the room such that the students are sitting in groups of about 4 at group tables. This allows them to share materials more easily.
- Give a copy of Template 2 to each student.
- Put a scissors or 2 on each group table.
- Put a roll of sellotape or 2 on each group table.
- Give each student a copy of the student instructions.
- Read the instructions aloud to the students before beginning the activity.

Template 2

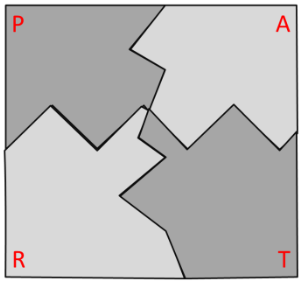


Student Instructions:

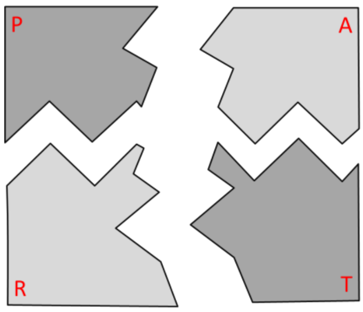
- Write the word “PART” in the corners of your square like the diagram below.
- Draw a random line on the square from left-to-right. It can be as squiggly as you want!!



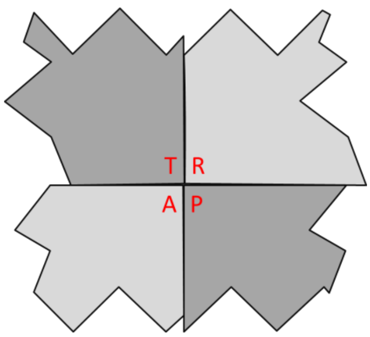
- Draw a second crazy line on the square from top-to-bottom.



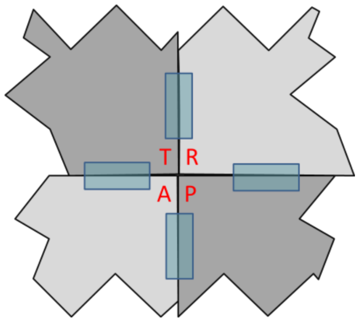
- Cut along the lines you drew to give you 4 shapes.



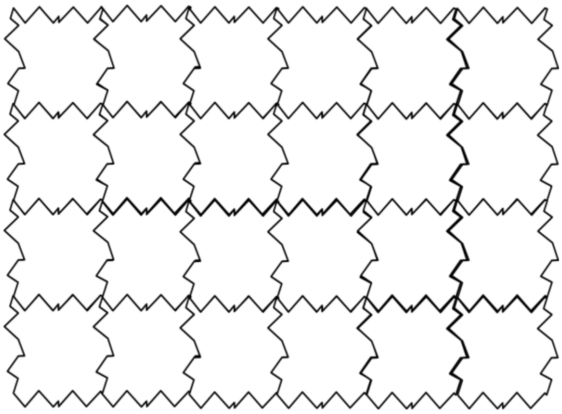
- Put the pieces together so that the corners are in the centre this time and they spell the word “TRAP” like this



- Tape the pieces together in this position and you've made a stencil that you can tessellate!!!



- Now fill up your page by tracing around the shape! If you followed all the steps correctly it should fit into itself perfectly!!!



Treat your stencil like a cloud...does it look like anything??????

Now colour it in!!!

In maths when we move a shape from one position to another in a straight line it is called a ***translation****. You just did lots of translations to fill up your page!!!*

Tessellations 2

Introduction: This lesson follows on from Tessellations 1. It focuses on symmetries and rotations.

Activity 1: Make your own Tessellation

Resources:

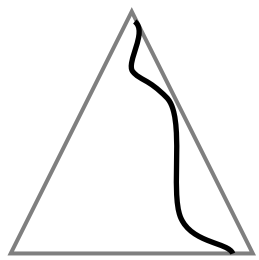
- Copy of Template 1 per student (printed onto card if possible).
- 2 sheets of card or thick paper per student
- 1 sheet of ordinary blank paper per student
- Scissors such that every student has access to one (Perhaps one to every 2 or 3 pupils)

Tutor instructions:

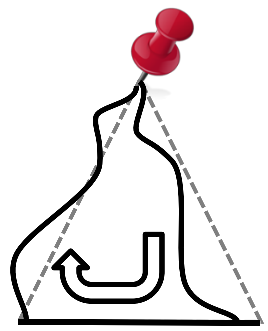
- This is an individual task where the students will be using a rotation method to create their own tessellation.
- Set up the room such that the students are sitting in groups of about 4 at group tables. This allows them to share materials more easily.
- Give a copy of Template 1 to each student.
- Put a scissors or 2 on each group table.
- Put a roll of sellotape or 2 on each group table.
- Give each student a copy of the student instructions.
- Read the instructions aloud to the students before beginning the activity.

Student instructions:

- Take your card triangle and make one side wavy like this



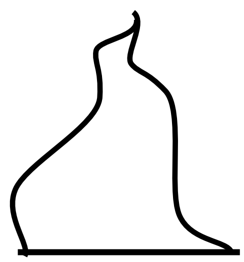
- Cut along your new line to give you a new triangle with 1 wavy side. Use this as a stencil. Place it down on a sheet of card and trace around it. Keeping it in place over the tracing hold down the top point of the triangle and swing it until the wavy line reaches the straight side of the one you've just drawn, like this...



(make sure the points of the triangle line up when you swing it into position)

When you swing your shape to line up with the straight side you are doing what we call a <u>rotation</u> in maths. Because it is an equilateral triangle the rotation is 60 degrees.

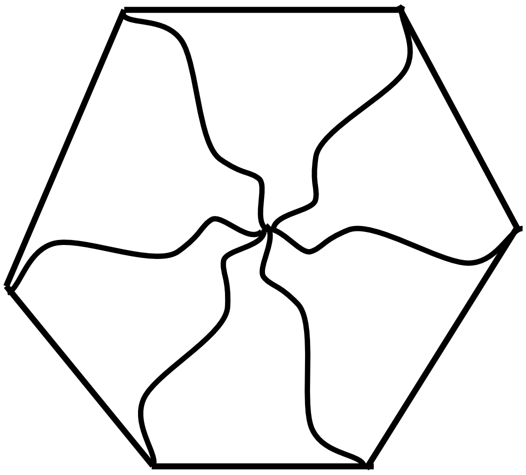
- Cut out your new triangle which now has 2 wavy sides. This is your stencil for your tessellation. Place your stencil on the blank page and trace around it. Then move your shape around so it fits next to itself again. Notice how it moves around almost like a circle.



Now colour in your design!!!

Solution:

The students work should look something like this:



Additional Activity: Escher Tessellation

Resources: (The students should have most of the resources for this activity)

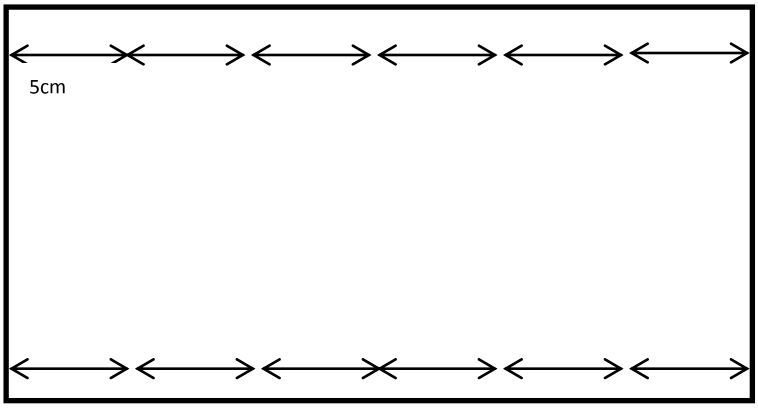
- Ruler per student
- Blank sheet of paper per student
- Pencil per student
- Eraser per student
- Copy of template 2 per student (this should be printed on card if possible)

Tutor instructions:

- This is an individual task where students will carry out a number of simple steps to make their own Escher style tessellation
- Give a blank sheet of paper to each student
- Ensure that every student has access to a ruler and eraser
- Give a copy of the student instructions to each student
- Read the student instructions aloud before beginning the activity

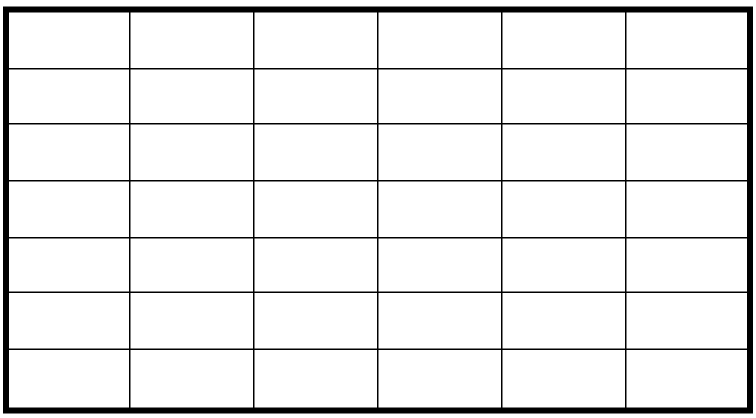
Student instructions:

- On your blank sheet of paper measure out and mark 5cm across the longer sides in pencil like so:

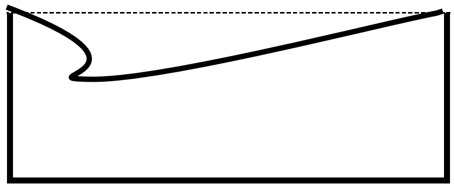


- Measure out and mark 2cm on the shorter sides in pencil like so:

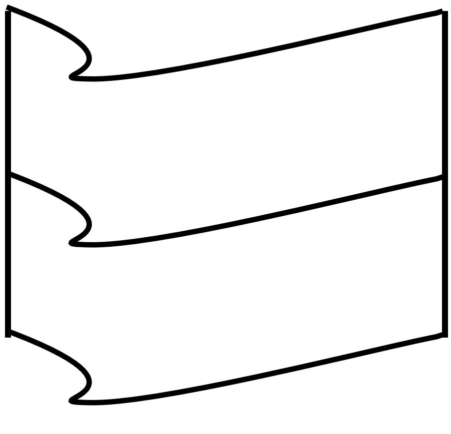
- Join these points in pencil to form a grid:



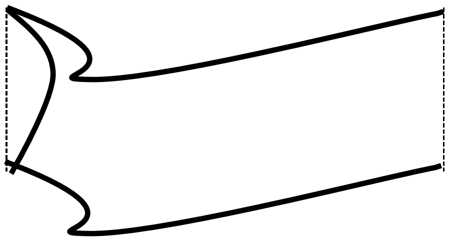
- On your cut-out template make a change to one of the sides of the rectangle and cut it out. The change can be whatever you want. Here's an example:



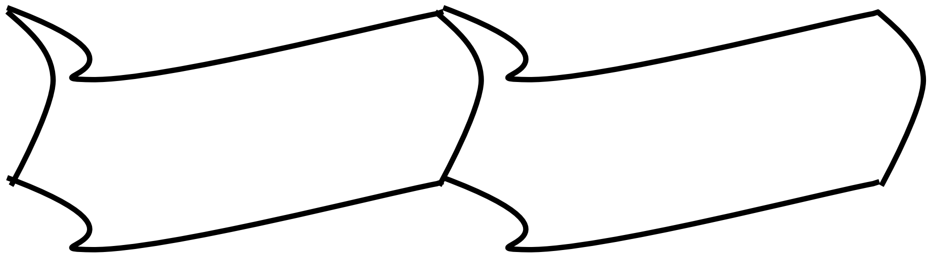
- Use your new shape as a stencil to change the look of a rectangle on your grid.
- Erase the original line of the rectangle in the grid and repeat your design for all the rectangles in the grid It will begin to look something like this:



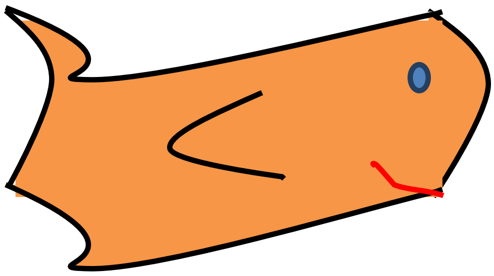
- Now repeat a similar process for the other sides of each rectangle. Make a change to the other sides of your stencil and use it to make changes to your grid:



It should start to look something like this:

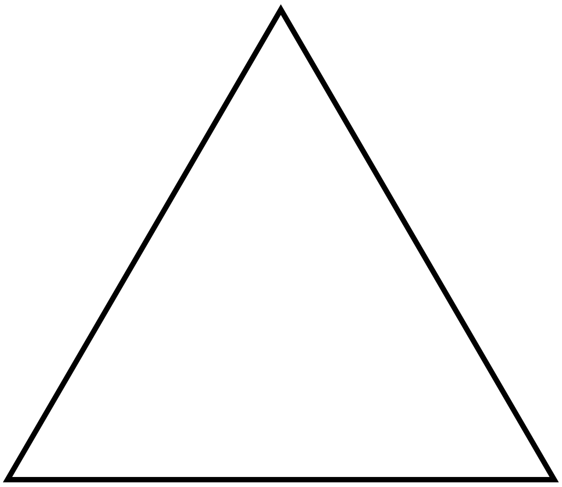


- Once you have made the changes to all the rectangles on your grid it's time to decide what your shape looks like. I think this one looks a little like a fish!

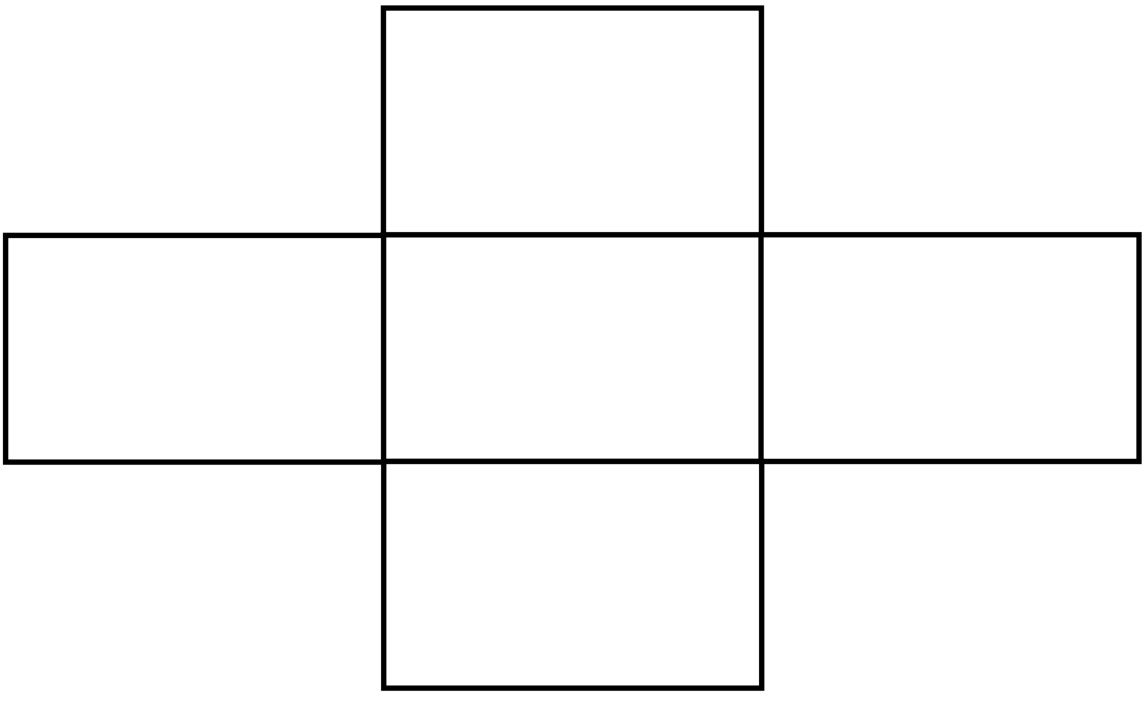


Now fill your page with whatever you think it looks like!

Template 1



Template 2



Tangrams, Origami and a Famous Theorem Lesson Plan 1:

Aims: To introduce basic shapes. To develop concepts of spatial reasoning/simple rotations/symmetry.

Online Game: http://pbskids.org/cyberchase/math-games/tanagram-game/

Materials Needed:

- Each student will need a sheet of A4 paper or light cardboard (needs to be foldable).
- Each student will need safety scissors.
- Print puzzle sheet from materials sheet at end. Need one per student.
- Possibility for teacher to make large Tangram set from wood/plastic depending on available resources.

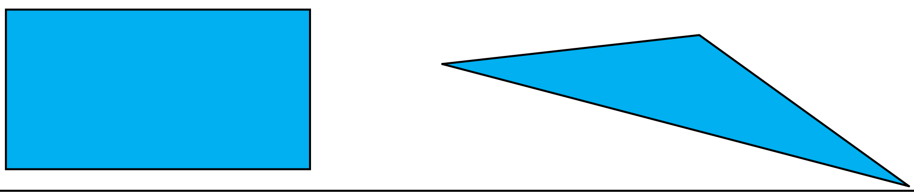
Introduction:

The **Tangram** is a dissection puzzle consisting of seven flat shapes, called *tans*, which are put together to form shapes. The aim of the puzzle is to form a specific shape (given only an outline or silhouette) using all seven pieces, which may not overlap. It was originally invented in China during the Song Dynasty, and then carried over to Europe by trading ships in the early 19th century.

Today we're going to make our own tangram set using **Origami**. Origami is the traditional Japanese art of paper folding, which started in the 17th century AD at the latest and was popularized outside of Japan in the mid-1900s. It has since then evolved into a modern art form and has been the subject of some interesting mathematical studies.

Before we begin let's talk about some basic shapes:

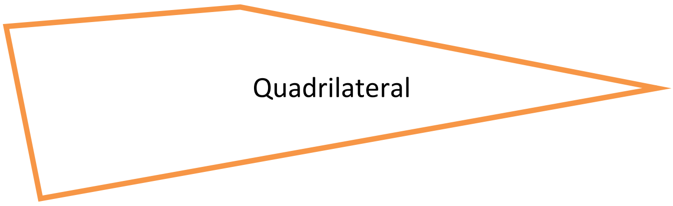
Group Discussion: What makes these two shapes different?



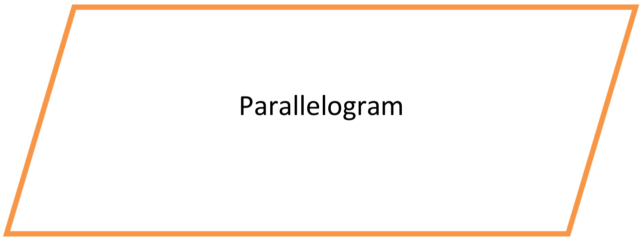
Group Discussion:

4 sided shapes:

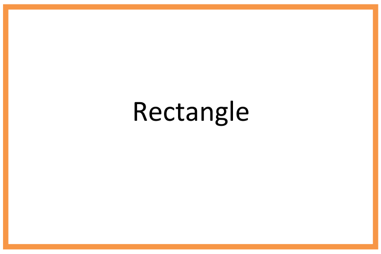
A four sided figure is called a **quadrilateral.**



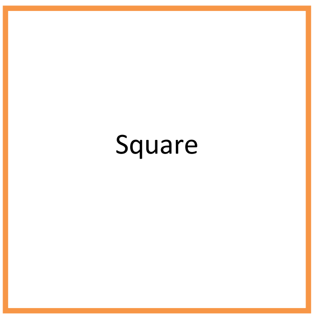
When opposite sides are parallel, we have a **parallelogram**. What does it mean for two lines to be parallel?



When all angles are right angles, we have a **rectangle**. What is a right angle?



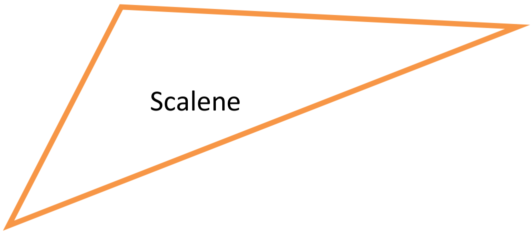
When all the angles are right angles and all the sides are the same length, we have a **square**.



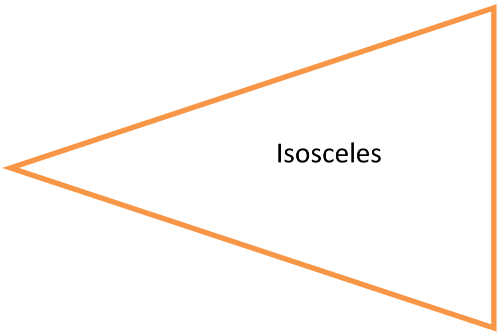
3 sided shapes:

A three sided figure is called a **triangle**.

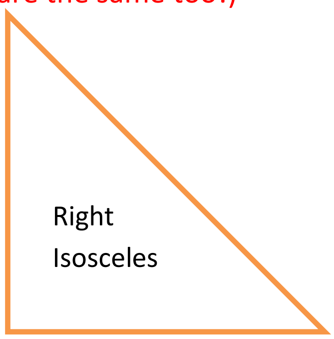
When all sides are different, we have a **scalene** triangle. (All angles are different too!)



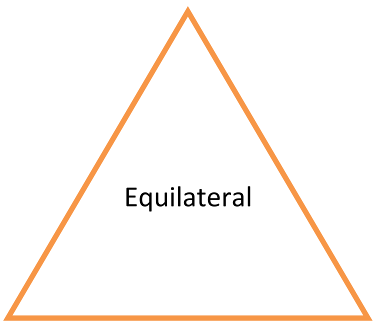
When two sides are the same we have an **isosceles** triangle. (Two angles are the same too!)



When two sides are the same and we have a right angle we have a **right isosceles** triangle. (Two angles are the same too!)



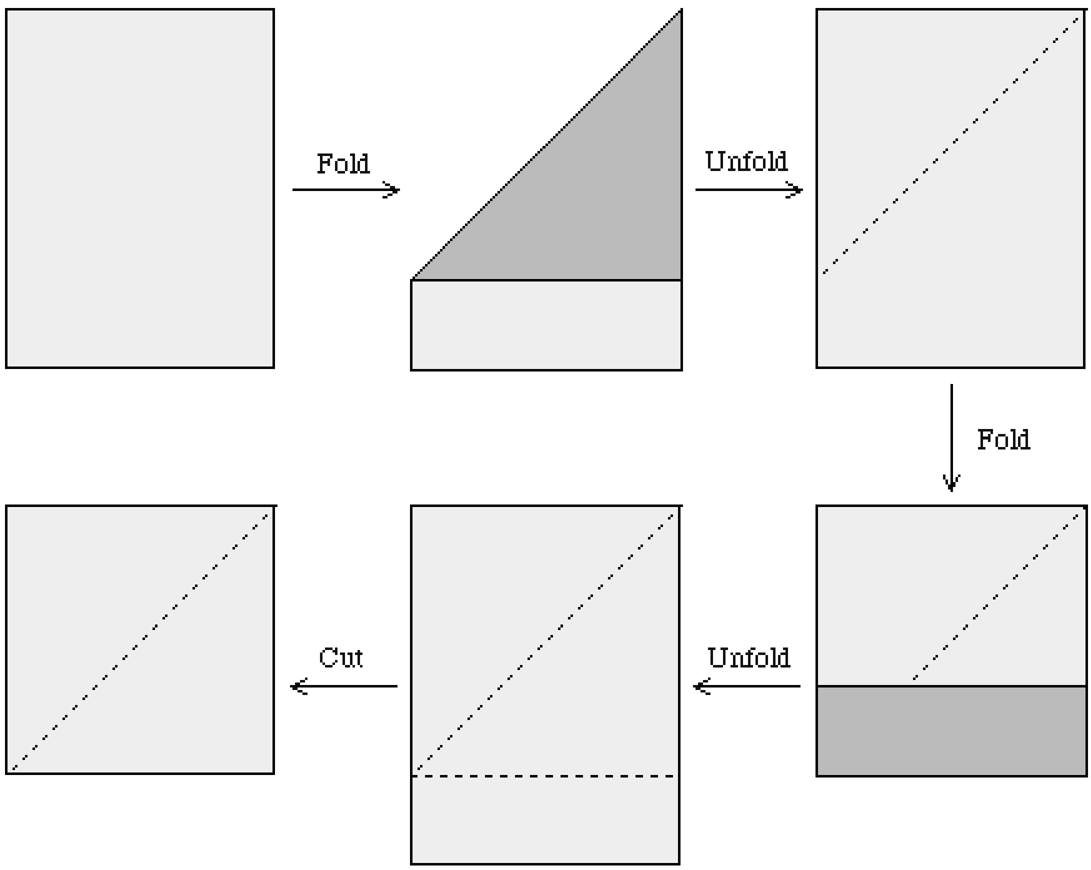
When all sides are the same, we have an **equilateral triangle.** (All angles are the same too)



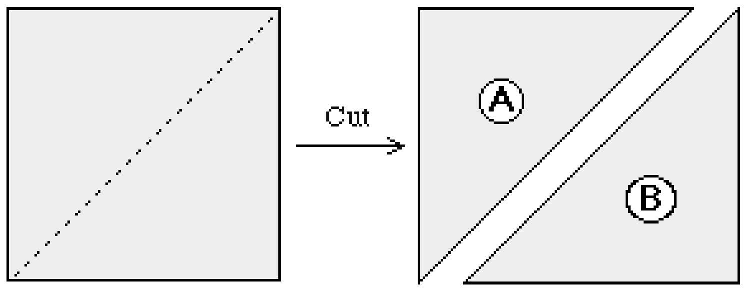
Question: What shapes are included in the Tangram Puzzle?

Individual Activity: Make your own Origami Tangram - Following the teacher. Each student has an A4 sheet and a safety scissors. Discuss progress at each step. What shapes remain? Why isosceles? Why are two shapes the same? Etc.

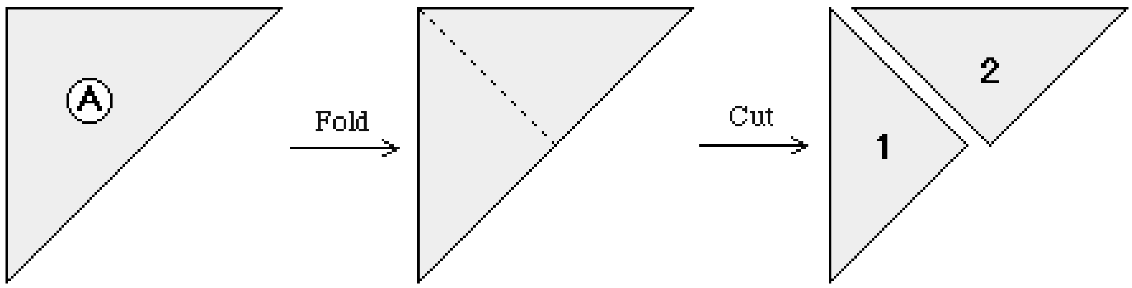
Step 1) What shape is the sheet of paper? Can we turn it into a square?



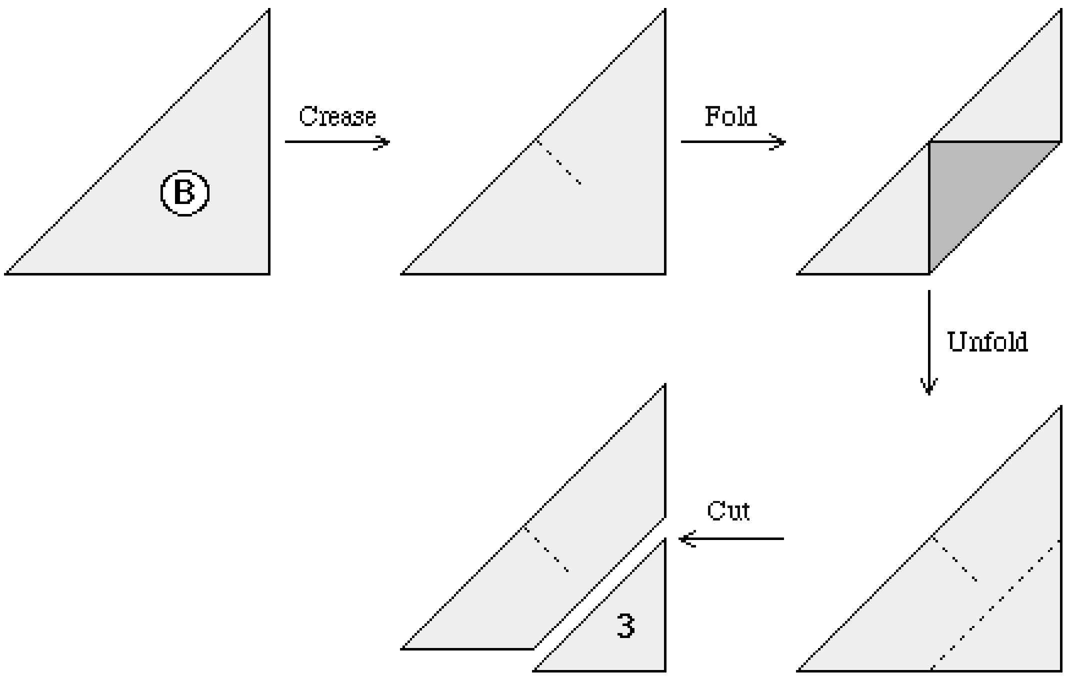
Step 2) Can we turn the square into two triangles? Cut along the fold line.



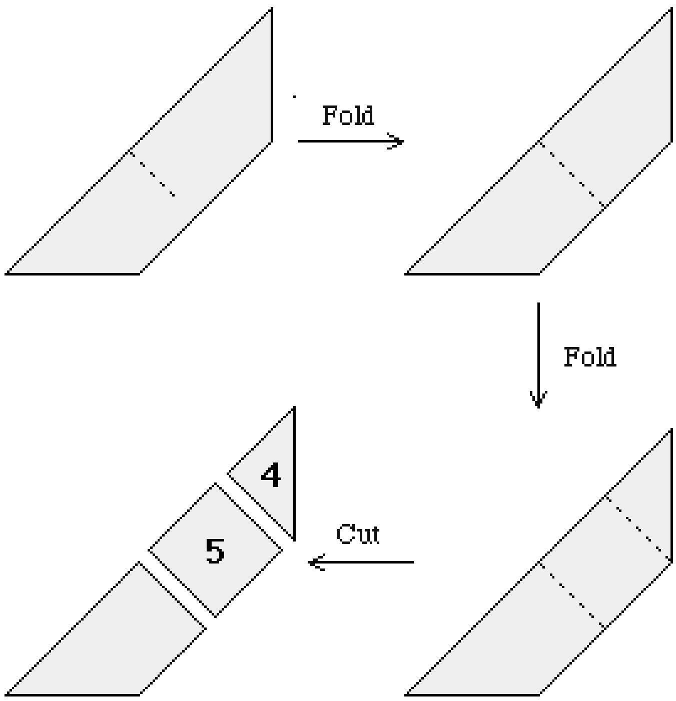
Step 3) Can we turn a right triangle into two right triangles? Take one triangle and fold it in half. Cut the triangle along the fold into two smaller triangles. Number them 1 and 2.



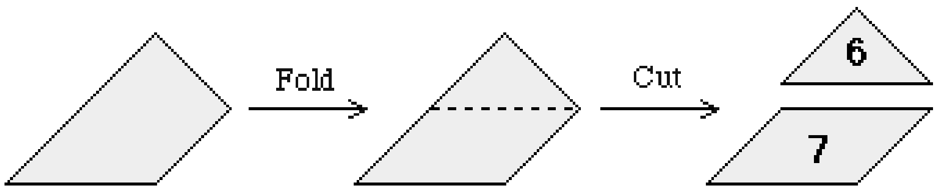
Step 4) What's left over? Take the other triangle and crease it in the middle. Fold the corner of the triangle opposite the crease and cut. Label it 3.



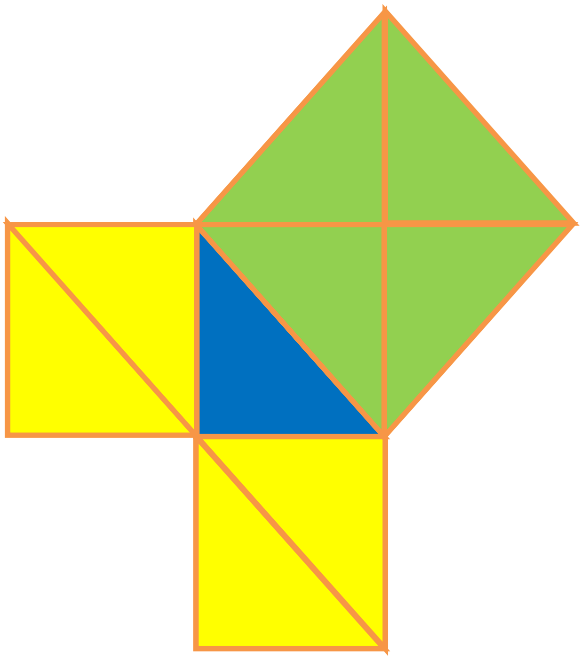
Step 5) Fold the remaining figure in half and fold again. Cut along both folds. Number the pieces 4 and 5.



Step 6) Fold the remaining figure and cut it in two. Number the pieces 6 and 7.



And we now have a Tangram puzzle!



Group Discussion/Activity:

Before we go puzzling, gather together 9 of the small right isosceles triangles that have label 1 or 2 and arrange them as in the picture. Do you notice anything?

If we build squares on each side of the blue triangle something very interesting happens. The green square built on the side opposite the right angle contains 4 small right isosceles triangles. Each little yellow square contains 2 small right isosceles triangles.

2+2=4 and the area of the big green square contains the same area as the two little squares combined. This is a special case of the **theorem of Pythagoras**. This theorem is one of the most famous results in mathematics and you'll see it again in secondary school. Hopefully this class will ring a bell!

Group Problems: (For Groups of 2/3) or individual competition, fastest to finish the puzzle wins. Distribute puzzle sheets among students.

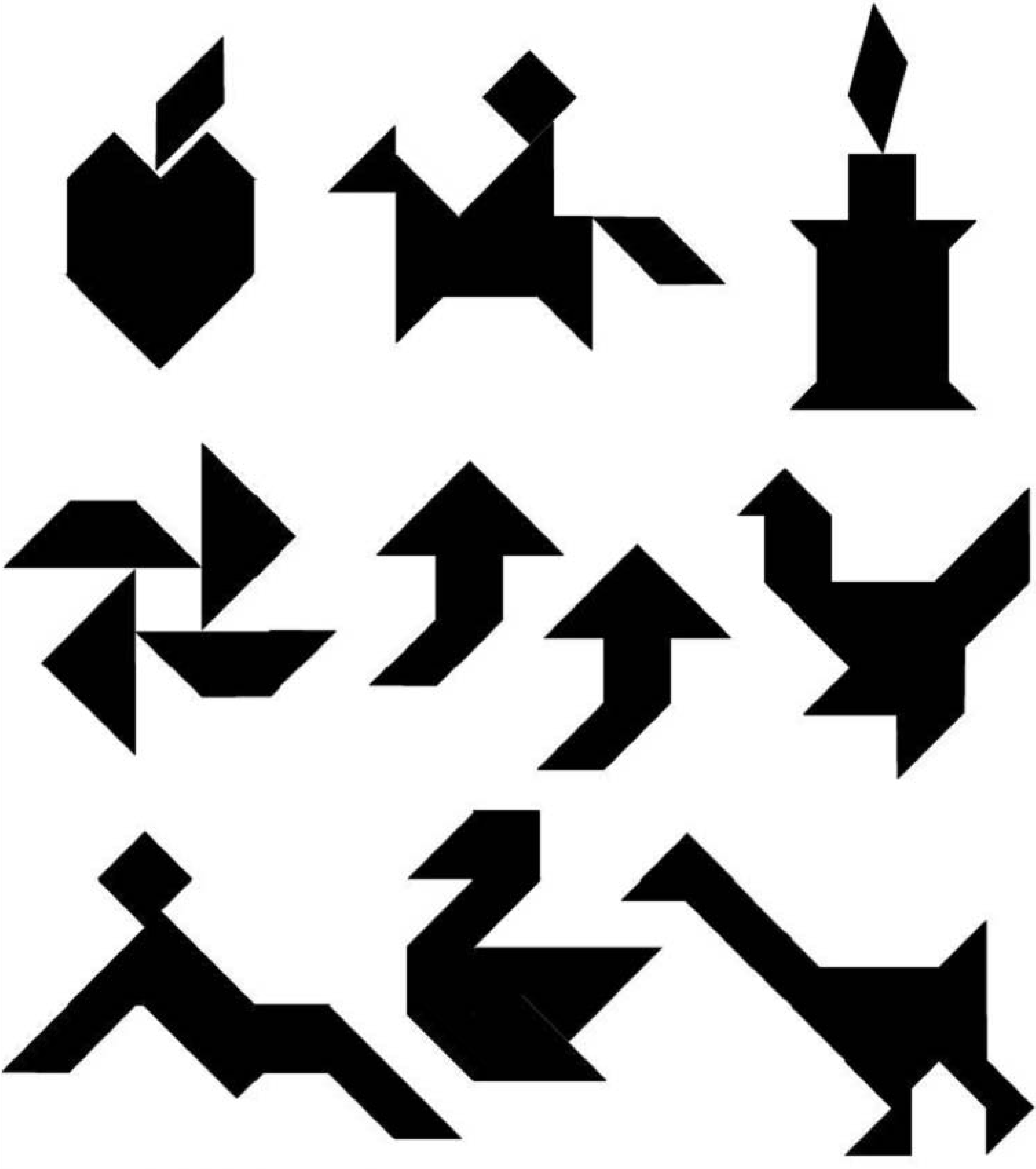
Try to make the following Tangram puzzles using the set we made. The rules are simple:

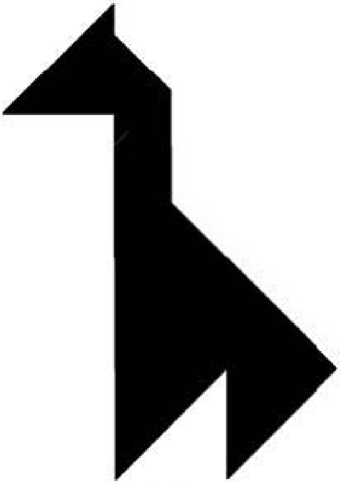
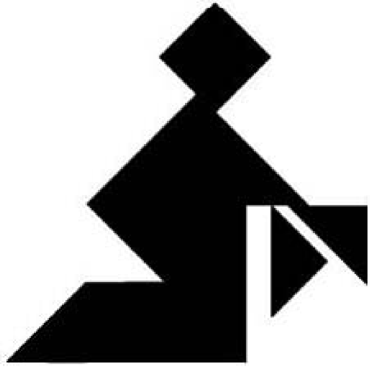
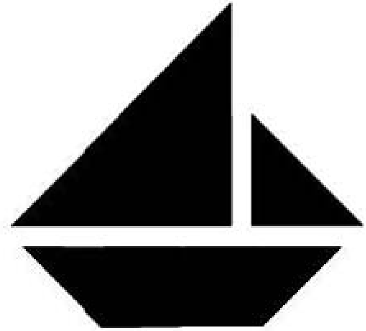
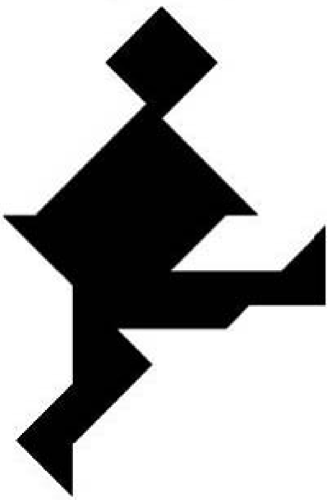
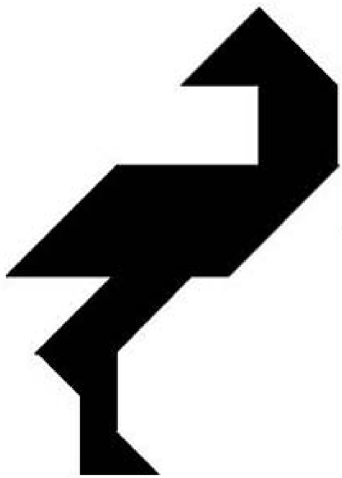
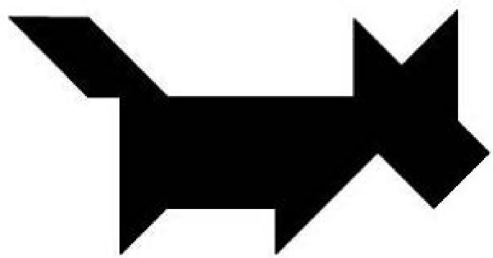
1. Use all 7 pieces.
2. Overlaps are not allowed.

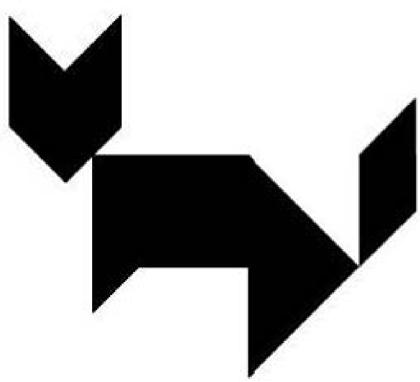
Take Home Problem:

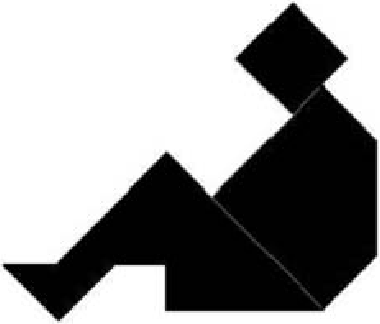
Using the Tangram set we made today, design your own puzzle. Use your imagination and be as creative as you like!

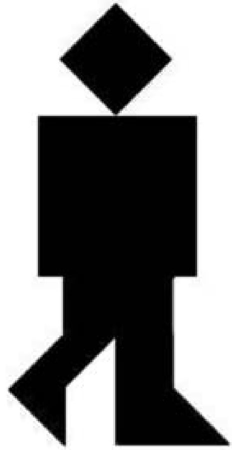
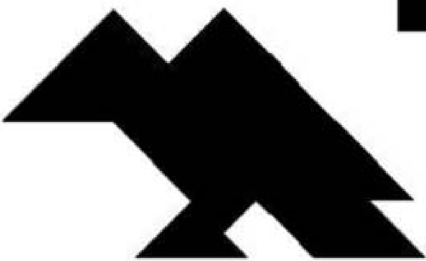
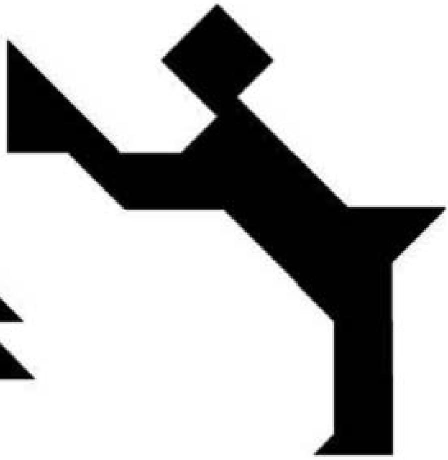
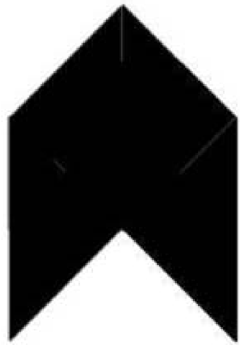
Solutions/Suggestions: Most questions are easily answered. The primary goal of this class is to familiarise students with different common shapes. Continual discussion should be carried out regarding number of angles/sides and number of equal angles/sides. Does the shape have a right angle? Etc.

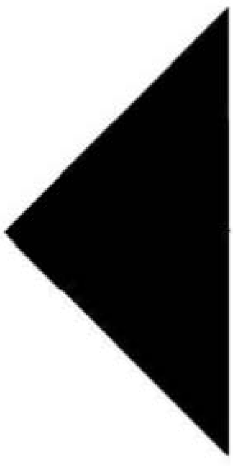









Sequences and Famous Fibonacci Lesson Plan 1:

Aims: To introduce simple sequences. To develop concepts of pattern and simple addition. Pattern occurs in nature too.

Materials Needed:

- Each student will need a worksheet from materials sheet at end.
- Matchsticks, glue and a large sheet of paper will be needed to make a Fibonacci spiral in groups of 2/3.
- Each group will need a compass or a piece of string and a pencil.

Introduction:

Much of what we know about the universe we've learned by indentifying patterns. Today we'll be looking at a particular type of mathematical pattern called a sequence.

A sequence is an ordered series of numbers. Once we understand the pattern behind the sequence, we should be able to figure out all the numbers in the sequence.

An very easy example is **1, 2, 3, 4, 5, 6, …** Can you guess what number comes next? What about after that?

Let's try a more difficult one: **2, 4, 6, 8, 10, 12, …** What's happening here? Between every two numbers, there's a jump of 2. What number comes next? What about after that? These numbers are called the **EVEN Numbers**.

Here's one more: **1, 3, 5, 7, 9, 11, …** What's happening here? Between every two numbers, there's again a jump of 2. What number comes next? What about after that? These numbers are called the **ODD Numbers**.

Group Activity/Discussion: Pick a starting number and a jump number. Move around the classroom forming the sequence as we go. Take note of the sequence on the board. Students should write the sequence down in their copybooks, taking note of the starting point and the jump. Do this 5 times.

Individual Activity: Distribute sequences exercise sheet. Each student should complete the sheet and paste into their copybook. The trick is to identify the jump.

The Fibonacci Sequence:

All the sequences we've seen so far follow a definite pattern. Between any two numbers there is a fixed jump. This is not the only way to define a sequence.

A very interesting and famous sequence was named after an Italian mathematician nicknamed Fibonacci. Fibonacci was one of the greatest European mathematicians of the middle ages. His full name was Leonardo of Pisa, or Leonardo Pisano in Italian since he was born in Pisa, the city with the famous Leaning Tower, in about 1175 AD.

This sequence even found its way into the best selling Dan Brown novel and film **The Da Vinci Code.**

The first few numbers in the famous sequence are

0, 1, 1, 2, 3, 5, 8, 13, 21, …

Group Activity/Discussion: Can we identify the pattern in the Fibonacci sequence? Remember that it's different to the sequences we've seen so far. What "rule" is being followed to get from one number to the next?

Can we figure out the next number? What about the next after that?

To get the Fibonacci numbers we start with begin the numbers 0 and 1. These are the first two Fibonacci numbers and the sequence looks like 0, 1.

To get the next number we add the two previous numbers together.

So, the next number is 0+1=1.

Our sequence now looks like 0, 1, 1.

Apply the rule again. The next number is 1+1=2.

So, our sequence now looks like 0, 1, 1, 2.

Apply the rule again. The next number is 1+2=3.

So our sequence now looks like 0, 1, 1, 2, 3..

We continue in the same way.

Fibonacci in Nature:

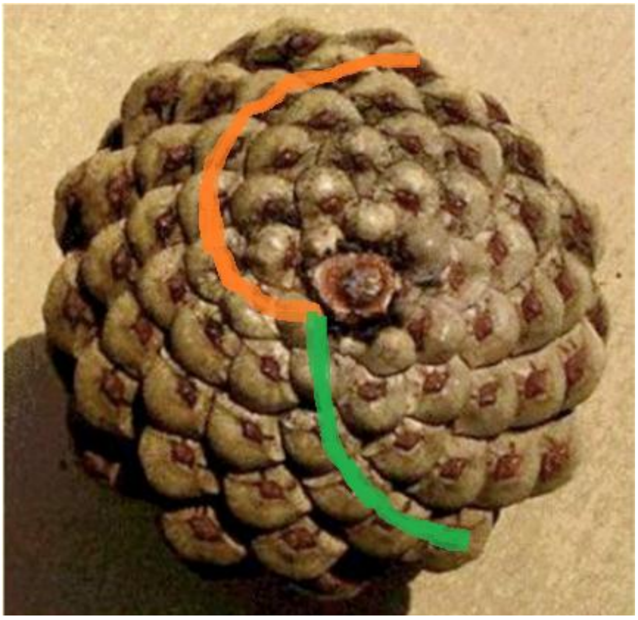
The Fibonacci numbers have intrigued mathematicians for centuries. What makes them so interesting is the fact that these numbers appear in many patterns in nature, often creating great beauty. Here's a nice example coming in the form of a pine cone.

On a pine cone, spirals form in two directions.

Individual Activity: In the exercise sheet, follow and draw in the spirals on the pine cone in both directions. Choose a different colour for each direction.

Two spirals are coloured in as a start. One direction is coloured orange. The other is coloured green.

Now, count the number of spirals in each direction. What do you notice?

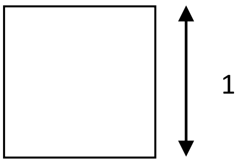


You should find 8 orange spirals and 13 green spirals. What's special about these numbers?

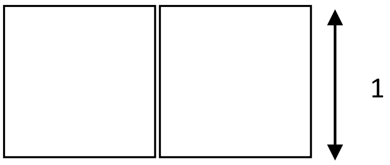
Group Activity (In groups of 2/3): Make your own Fibonacci spiral. Students need matchsticks, an A3 sheet of paper and some glue. The best strategy here is to first make the spiral on a desk and then transfer it to a sheet of paper when you are happy with the design.

We are going to make different sized squares with the matchsticks. The squares will have sides whose lengths are consecutive Fibonacci numbers. When we say a square with side 1, we mean that we use 1 matchstick for each side and so on.

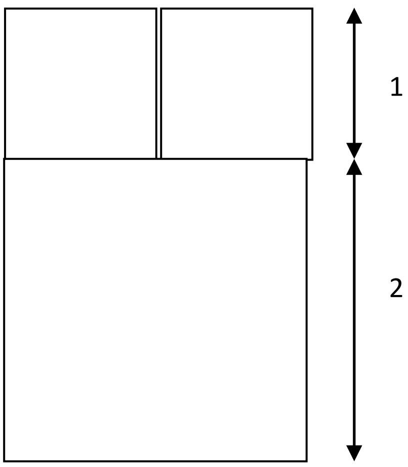
Step 1: The first Fibonacci number is 1. Make a square of side 1 with the matchsticks.



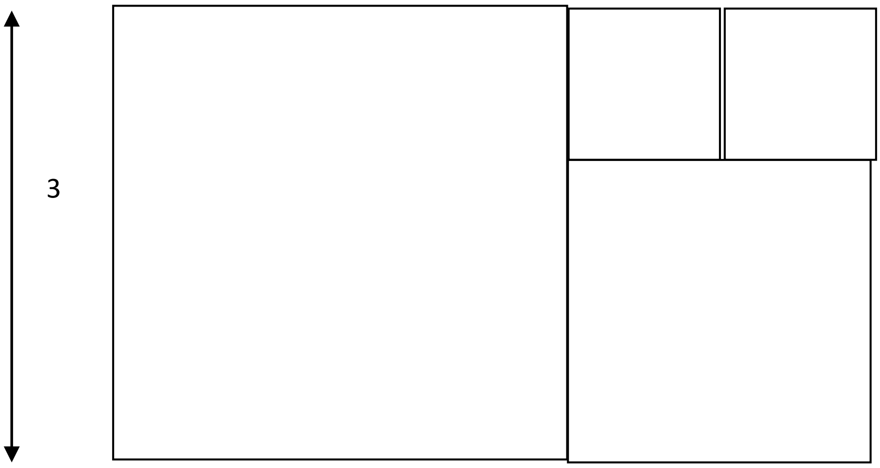
Step 2: The next Fibonacci number is 1. Make a square of side 1 with the matchsticks. Join it to the first square we made.



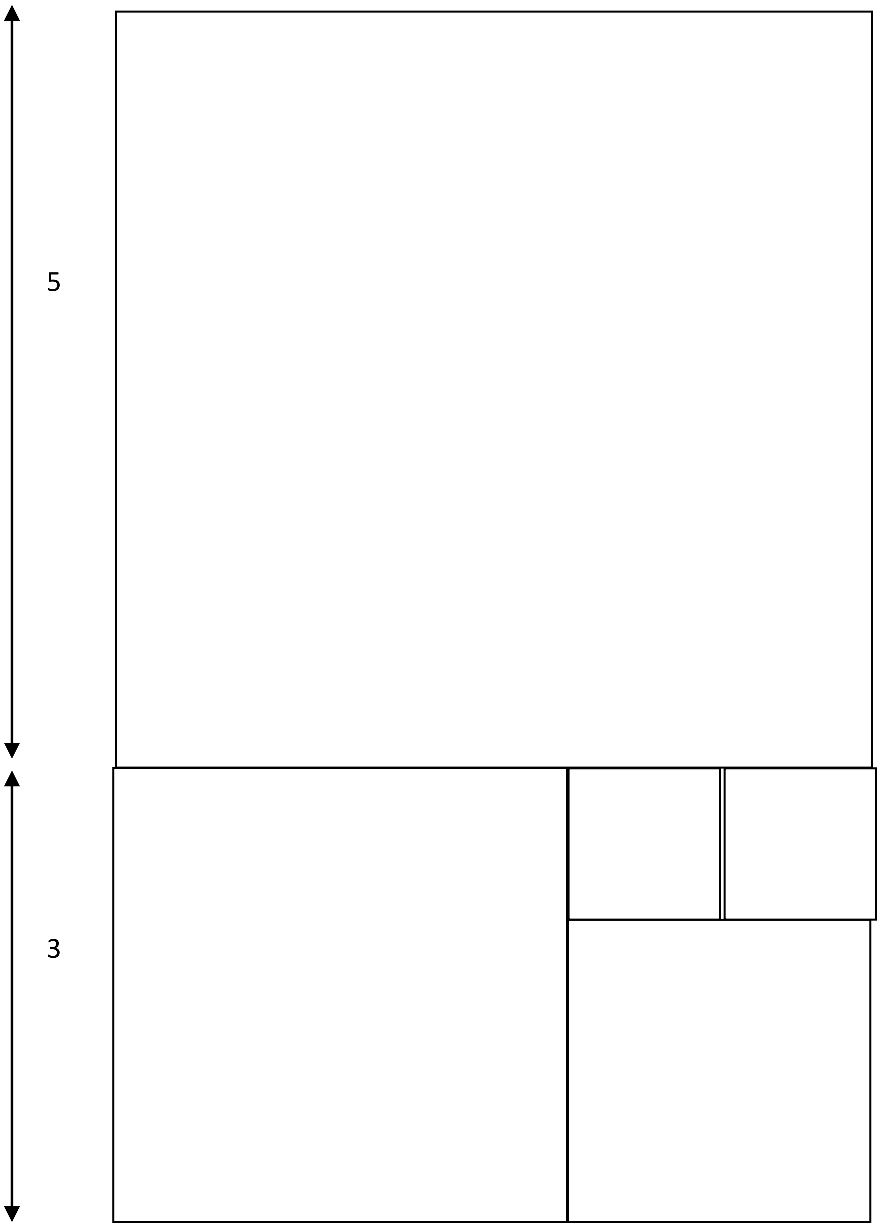
Step 3: The next Fibonacci number is 2. Make a square of side 2 with the matchsticks. Join it to the shape have we made at Step 2.



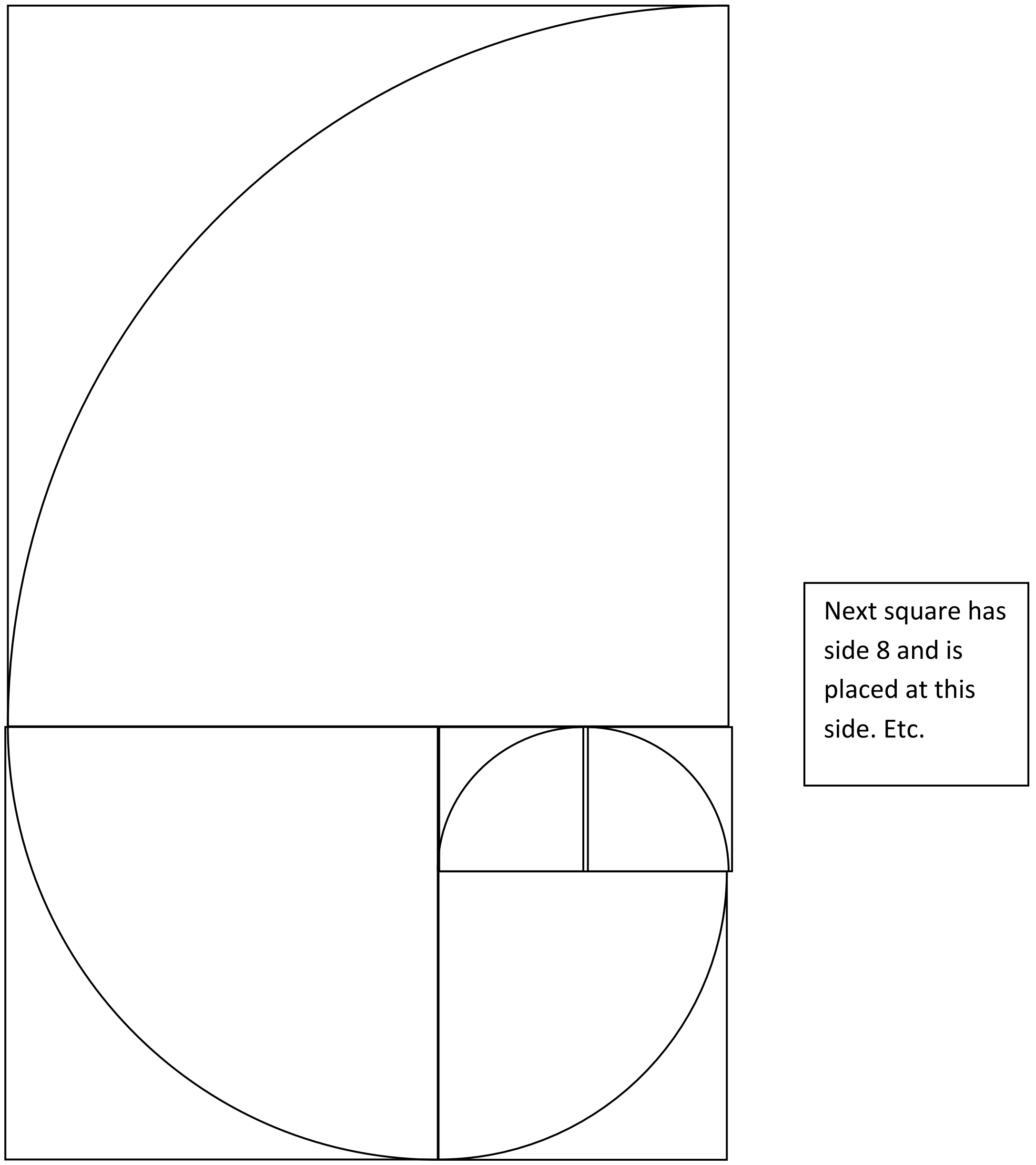
Step 4: The next Fibonacci number is 3. Make a square of side 3 with the matchsticks. Join it to the shape have we made at Step 3.



Step 5: The next Fibonacci number is 5. Make a square of side 5 with the matchsticks. Join it to the shape have we made at Step 4.



Step 6: Let's see how we're doing. Using a compass or a string and a pencil, trace out the arcs as we see below. There's a pretty picture forming.



Step 7: By now, you probably get the picture. At each step add on a new square with side the length of the next Fibonacci number. Seeing how the spiral is forming gives a clue as to what side to place the square. Keep repeating the steps until you run out of space.

Step 8: If you are happy with your spiral, you can glue the matchsticks to a large sheet of paper. Colour it in and decorate as you like. Does it remind you of anything?

A Fibonacci spiral on a nautilus shell:

Take home problem: Make your own Fibonacci sequence. Pick your two favourite numbers. These will be the starting point for building your very own Fibonacci sequence.

Remember the rule: You get the next number by adding together the two previous numbers.

Write out the first 10 numbers in your Fibonacci sequence.

Materials Sheet: Print one per student:

Fill in the blanks in the following sequences:

- 3, 6, 9, 12, 15, 18, __, __ Jump = __
- 4, 8, 12, 16, 20, 24, __, __ Jump = __
- 5, 10, 15, __, 25, __, 35, 40 Jump = __
- 10, 20, 30, __, __, 60, 70, 80 Jump = __
- 3, 5, 7, 9, 11, 13, __, __ Jump = __
- 5, 8, 11, 14, 17, 20, __, __ Jump = __
- 6, 12, __, 24, 30, 36, __, 48 Jump = __
- __, __,7, 10, 13, 16, __, 22 Jump = __
- 8, __, 22, 29, 36, 43, __ Jump = __

Pine Cone Exercise:

Tetris, Polyominos and Tiling Lesson Plan 1:

Aims: To develop concepts of spatial reasoning/simple rotations/area/perimeter/symmetry.

Materials Needed:

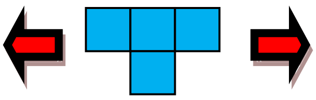
- Students will need a sum copybook and a plastic pocket to keep cutouts in.
- Each student will need safety scissors.
- Each student will need a grid sheet printout from materials sheet at end.
- Print and ideally laminate chessboards from materials sheet at end. Need one of each for each group of 2/3 students. **These are reusable.**

Introduction:

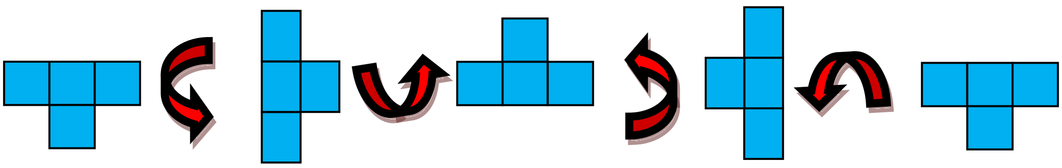
Most people are familiar with the video game **Tetris**. The playing pieces are geometric shapes consisting of four blocks each called **Tetrominos** which fall randomly down the playing field. The playing field is 10 blocks wide.

The playing pieces can be moved in two ways as they fall;

- Pieces can be moved horizontally.



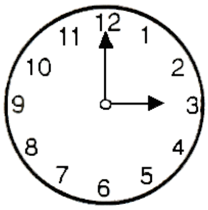
- Pieces can be rotated by 90 degrees at time.



The angle 90 degrees is a special angle called a **right angle.**

It's the angle between the hands of a clock when the time is 3 o' clock.

Think of a right angle as a perfectly square corner.



Group Activity/Discussion:

How many right angles can you spot in the classroom?

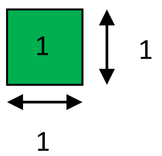
Write some down in your copybook.

The aim of Tetris is to create horizontal lines of ten blocks without any gaps. When a full line is formed, it disappears and any block above the deleted line falls downwards. The more lines cleared the faster the playing pieces fall. The game ends when the stack of tetrominos reaches the top of the playing field and no more can fall.

Aside from the game being fun and very addictive, Tetris is also very interesting from a mathematical point of view. The playing pieces come from a family of shapes called **Polyominos**. The simplest polyomino is called the monomino. We can think of a monomino simply as a square with each side being 1 unit long and having an area of 1 square unit.

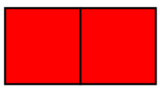
Remember: Area is the amount of space contained within a shape. You might remember the formula Area = Length x Width. Perimeter is the distance around the outline. A monomino has a perimeter of 4 units and an area of 1 square unit.

Here is the humble monomino. We must distinguish between the 1's. The 1's on the edges are lengths. The 1 in the middle represents an area and is measured in square units.

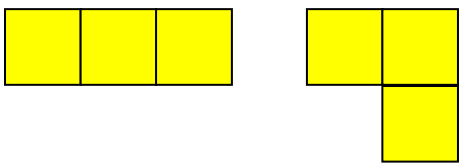


Monominos on their own look quite uninteresting but when we join them together along edges things get much more interesting. We get what are called **free polyominos**. By **free** we mean that we can move them left and right, rotate by 90 degrees as many times as we like and flip them over.

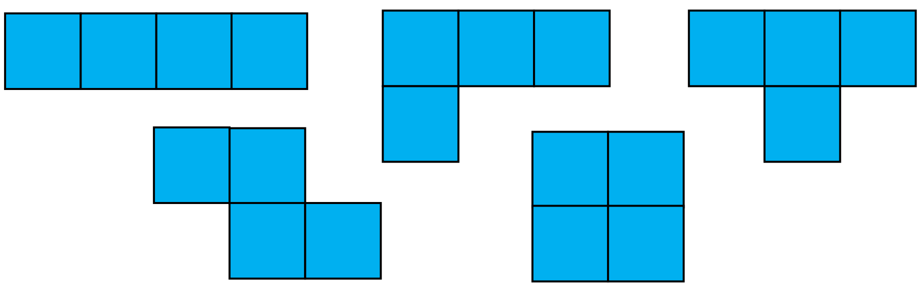
2 Monominos = 1 Domino. There is one of these:



3 Monominos = 1 Tromino. There are two of these:



4 Monominos = 1 Tetromino. There are five of these:



Individual Activity:

In your sum copybook, make a copy of the monomino, the domino, the 2 trominos and the 5 tetrominos shown above. Make the monomino the size of 1 box.

Now imagine that the tetrominos are fields that we want to fence in. How much fencing do we need to fence each one? Which field is the biggest? In each case, are we talking about area or perimeter?

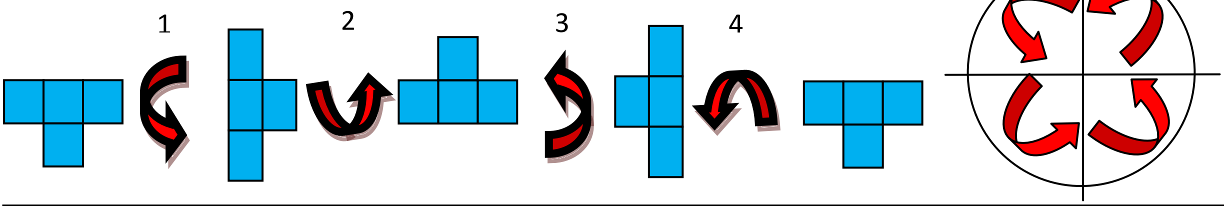
Individual Activity:

Using the large grid sheet, make copies of all the polyominos we have seen so far.

Cut them out using safety scissors and arrange them in their groups.

Colour each group a different colour.

4 rotations by 90 degrees make up a full circle. So, if we do 4 rotations in a row, we are always back where we started like we see below.

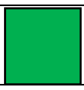


Individual Activity:

For each polyomino cutout, how many 90 degree rotations does it take us to get back to where we started? Write down the numbers in your copybook.

Hint: Some will be less than 4.

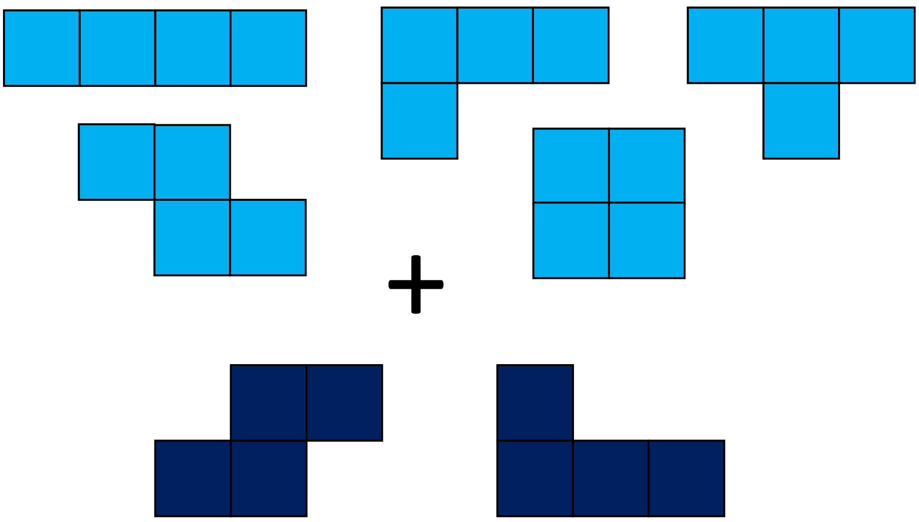
Make and fill in a table in you sum copy that looks like this. The first row is filled in below.

Shape (Picture)	**Number of blocks**	**Perimeter**	**Area**	**Number of rotations to get back to start**
	1	4	1	1

Group Activity/Discussion

In Tetris, there are in fact seven playing pieces. Why are the two extra ones included?

Hint: What moves are allowed in Tetris? How can we get the extra pieces starting from the 5 tetrominos we began with?

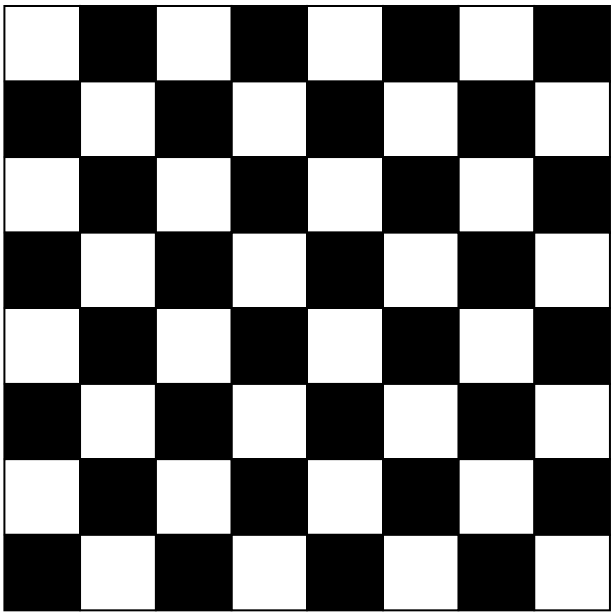


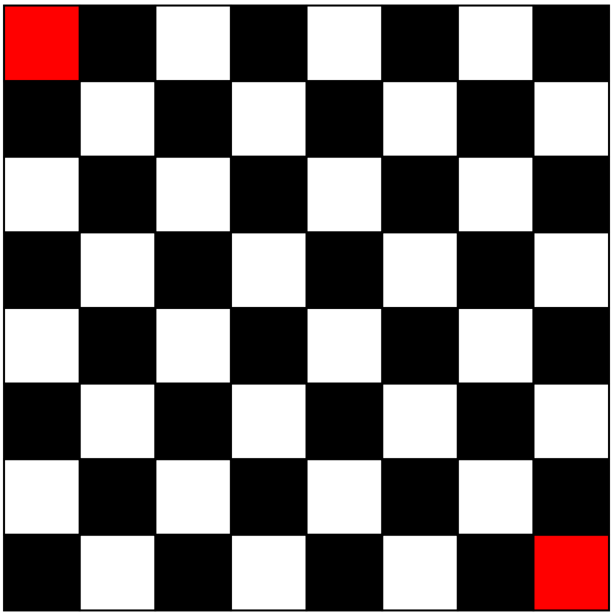
Group Domino Problems: (For Groups of 2/3) Using laminated chessboards and domino cutouts.

We might need to cut out some more dominos!!!

Here’s a chessboard. It is 8 units long and 8 units wide.

1. How many monominos does it take to cover the chessboard?
2. Can we cover the chessboard with dominos? If so, show a tiling. How many must dominos must we use?





Let’s see what happens if cut off two diagonally opposite corners of the chessboard.

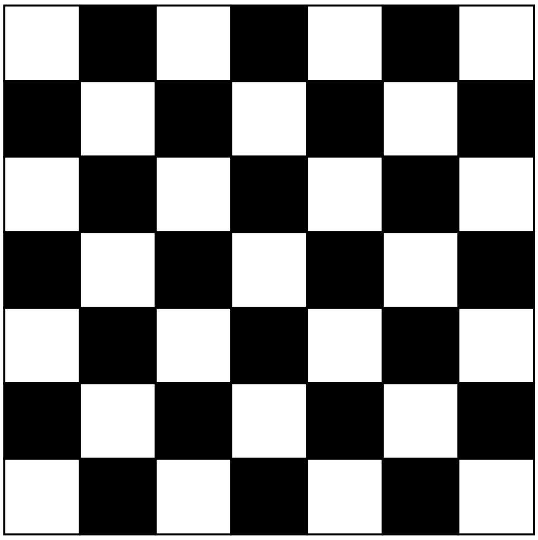
Problem: Is it still possible to cover the chessboard with dominos? If so, show a tiling. If not, why not?

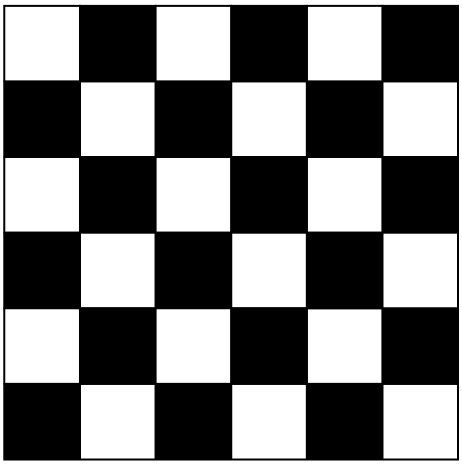
Let’s experiment with different size chessboards.

7 units long and 7 units wide

Questions:

1. How many monominos does it take to cover the chessboard?
2. Can we cover the chessboard with dominos? If so, show a tiling. If not, why not?
3. What if we mix dominos and monominos? Can we cover the board now? What’s the most dominos we can use?





6 units long and 6 units wide

1. How many monominos does it take to cover the chessboard?
2. Can we cover the chessboard with dominos? If so, show a tiling. If not, why not?

Final Group Discussion:

What boards were we able to tile perfectly using dominos?

Generally, what sizes of square board do you think can we cover with dominos?

Take Home Problem:

Using all of the polyomino cutouts we have made so far, arrange them to make a pretty picture and paste it to a sheet of paper. Use your imagination and be as creative as you like!

Solutions:

Question: Imagine the tetrominos are fields and we want to fence them. How much fencing do we need to cover each one? Which field is biggest?

Answer: They all need a fence of length 10 except for the square tetromino which needs a fence of length 8. All the fields have the same size, just different shapes.

Question: For each polyomino cutout, how many 90 degree rotations does it take us to get back to where we started?

Answer: The possibilities are 1, 2 or 4. The "square" polyominos are unaffected by rotation so are returned to original position after 1 or no rotation. Any "straight line" polyomino will be back its original place after 2 rotations. The rest take 4 rotations.

Question: In Tetris, there are in fact seven playing pieces. Why are the two extra ones included?

Answer: In Tetris, the allowed moves are rotations by 90 degrees, 180 degrees and 270 degrees and moves to the left or right. We are not allowed to flip pieces over. So the two "flipped" pieces are also included.

For the 8 by 8 Chessboard:

1. How many monominos does it take to cover the chessboard? **Answer:** 64 monominos
2. Can we cover the chessboard with dominos? If so, show a tiling. How many must dominos must we use? **Answer:** 32 dominos, easy to show.

For the 8 by 8 Chessboard with Opposite Diagonals Removed:

1. Is it still possible to cover the chessboard with dominos? If so, show a tiling. If not, why not? **Answer:** It's impossible. Each domino covers two squares, a white and a black one. We're removing two white, so there's 32 black and 30 white left and we can't cover the board.

For the 7 by 7 Chessboard:

1. How many monominos does it take to cover the chessboard? **Answer: 49**
2. Can we cover the chessboard with dominos? If so, show a tiling. If not, why not? **Answer:** No! There will always be one square left over.
3. What if we mix dominos and monominos? Can we cover the board now? What's the most dominos we can use? **Answer:** Yes. The board above has white corners. There is an extra white square if we count up the different white and black squares. Put the monomino on a white square and the rest is easy.

For the 6 by 6 Chessboard:

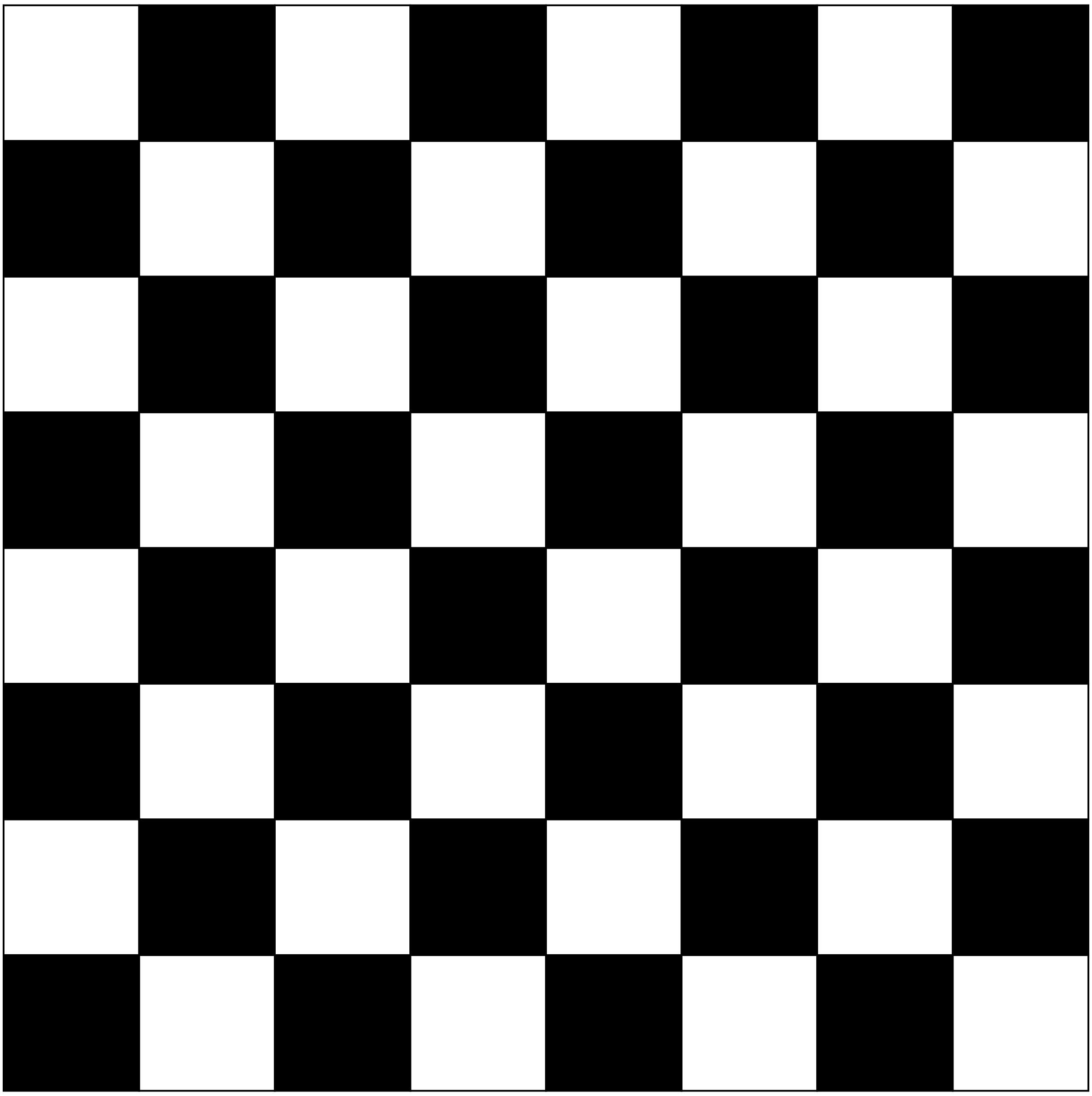
1. How many monominos does it take to cover the chessboard? **Answer:** 36 monominos
2. Can we cover the chessboard with dominos? If so, show a tiling. How many must dominos must we use? **Answer:** 18 dominos, easy to show.

Question: What sizes of square board can we cover with dominos? **Answer:** If the board has sides of even length, its area will be even too. Then it's easy to cover with dominos.

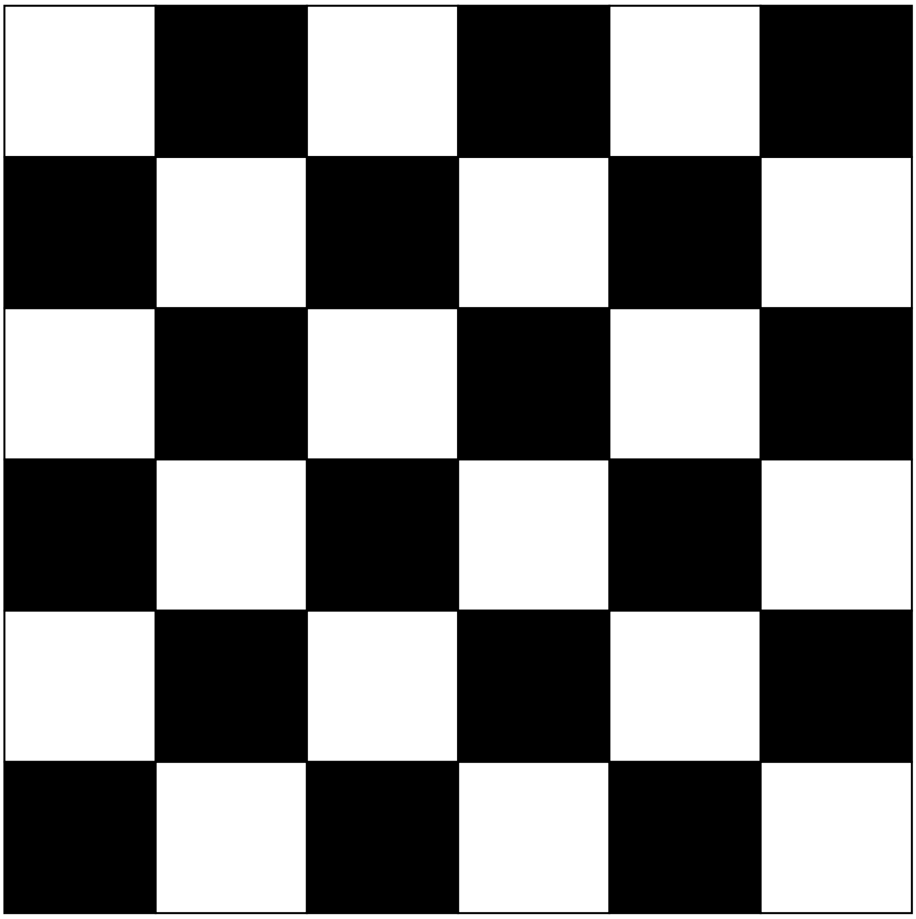
Facts: Even * Even = Even and Odd * Odd =Odd

Grid sheet. Print 1 per student.

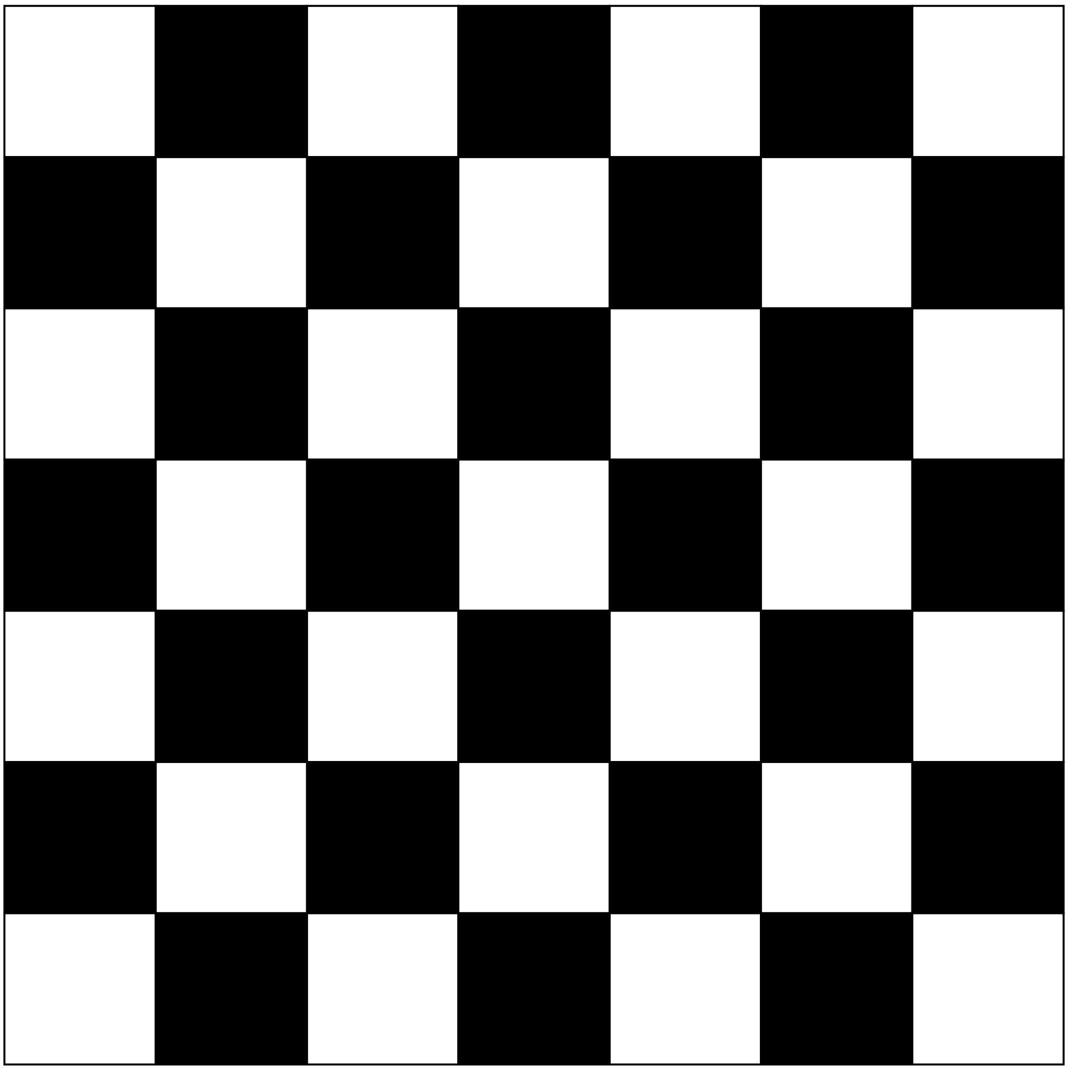
Chess Boards: Print and laminate one of each per group of 2/3 students.



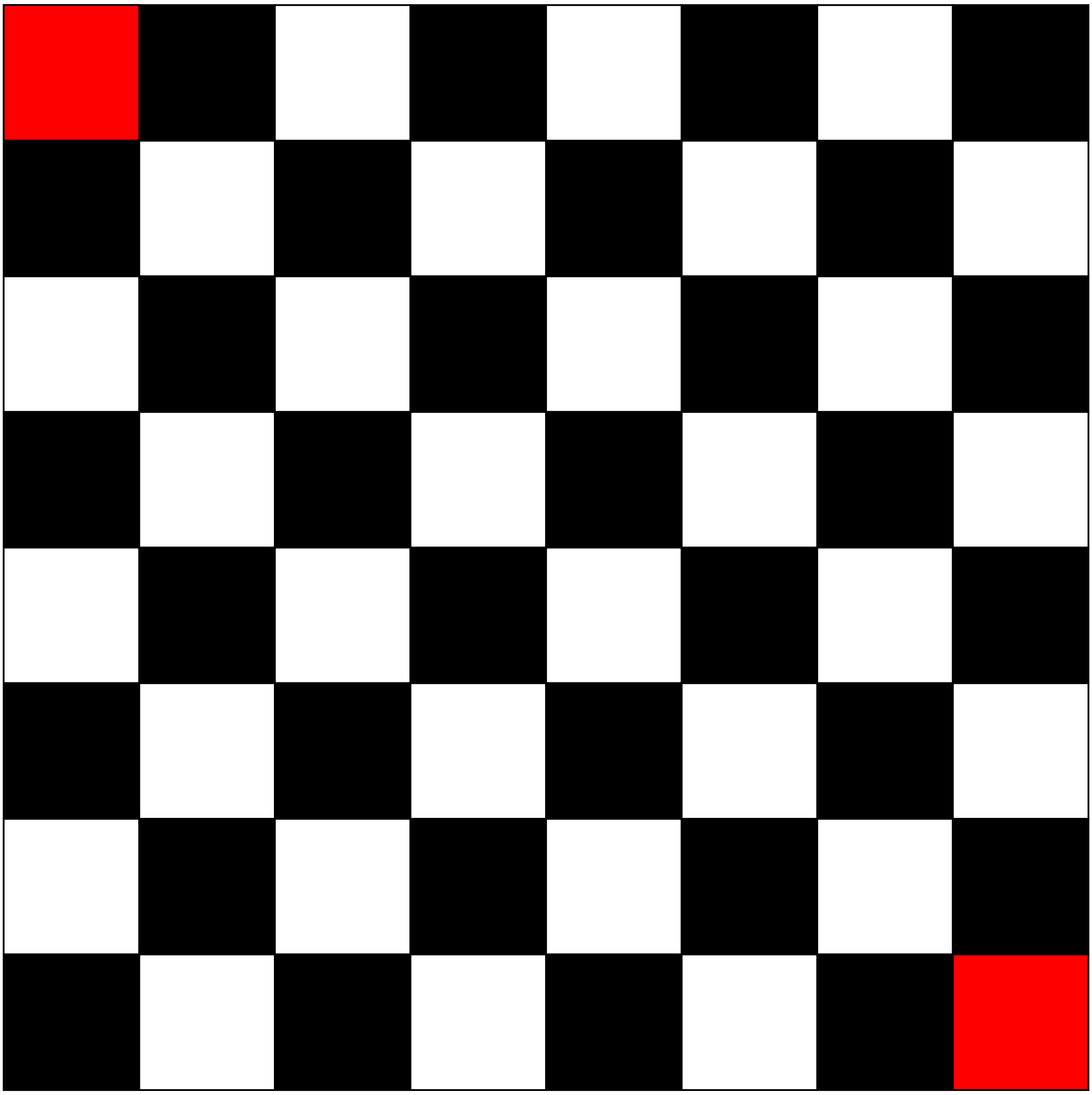
8 by 8 Chessboard



6 by 6 Chessboard

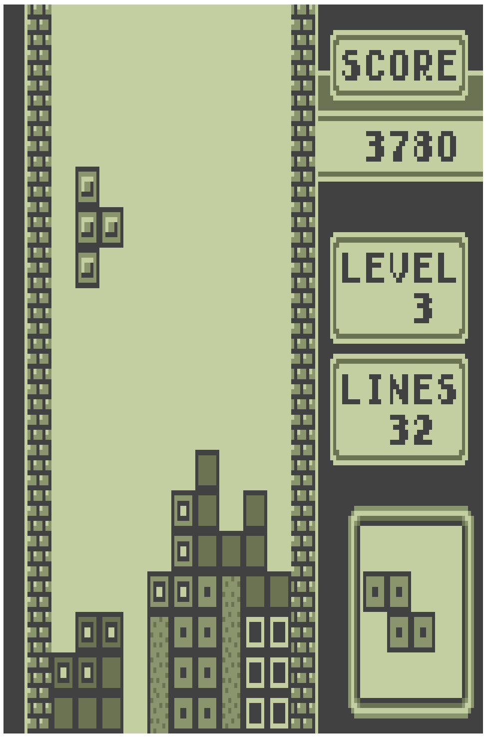


7 by 7 Chessboard



8 by 8 chessboard with opposite corners removed

Tetris, Polyominos and Tiling Lesson Plan 2:

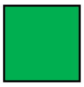


Aims: To develop concepts of spatial reasoning/simple rotations/area/perimeter/symmetry/basic multiplication/factors.

Materials Needed:

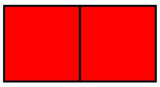
- Students will need a sum copybook and a plastic pocket to keep cutouts in.
- Each student will need safety scissors.
- Each student will need a pentominos grid sheet printout from materials sheet at end.
- Print and ideally laminate grids from materials sheet at end. Need one of each for each group of 2/3 students. **These are reusable.**

Introduction:

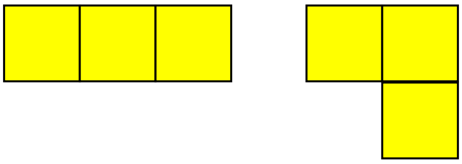
Remember the humble monomino: 

When we join monominos together along edges, things get a lot more interesting.

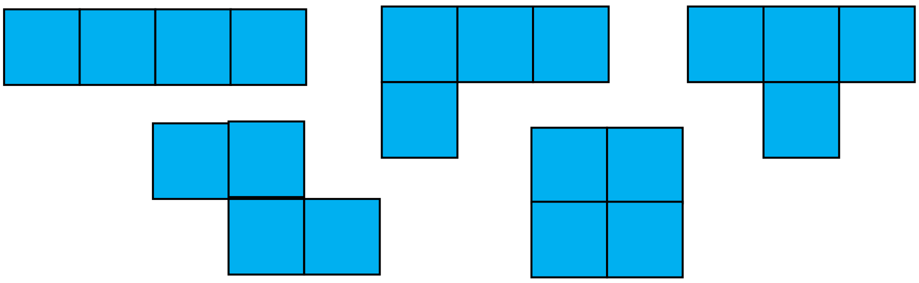
2 Monominos = 1 Domino. There are is one of these:



3 Monominos = 1 Tromino. There are two of these again:



4 Monominos = 1 Tetromino. There are five of these:

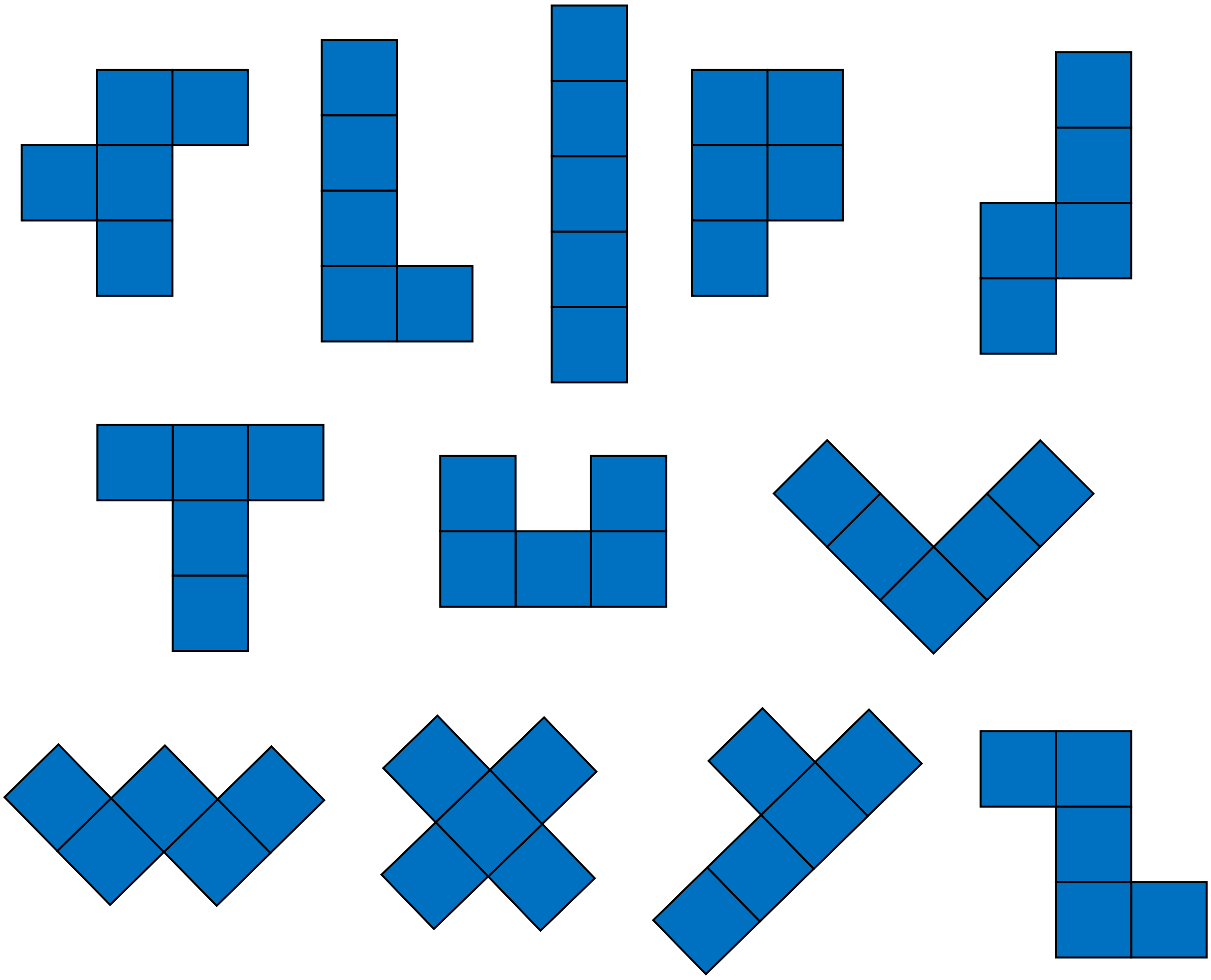


Individual Challenge/Group Discussion:

5 Monominos = 1 Pentomino. How many pentominos can you find? Draw them in your copybook as you discover them. **Remember, they must be joined along edges.**

Remember also that we can rotate them and flip them over so we may find some that are essentially the same as others.

The 12 Free Pentominos (Free means we can flip them over!)



There's an easy way of naming the pentominos. Each one "looks" like a letter. We have to use our imaginations for some. As they are arranged here, they read:

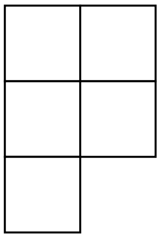
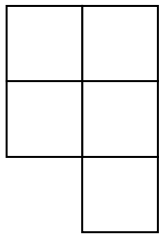
F-L-I-P-N-T-U-V-W-X-Y-Z

Individual Activity:

In your sum copybook, draw a copy of the 12 pentominos shown above. How many did you find?

Now imagine that the pentominos are fields that we want to fence in. How much fencing do we need to fence each one? Which field is the biggest? In each case, are we talking about area or perimeter?

We shall use one more symbol: When we place a star (*) on a letter that denotes a pentomino, we mean that we shall need to use the reverse side of the pentomino. For example, P* denotes the pentomino P after we have reversed it.

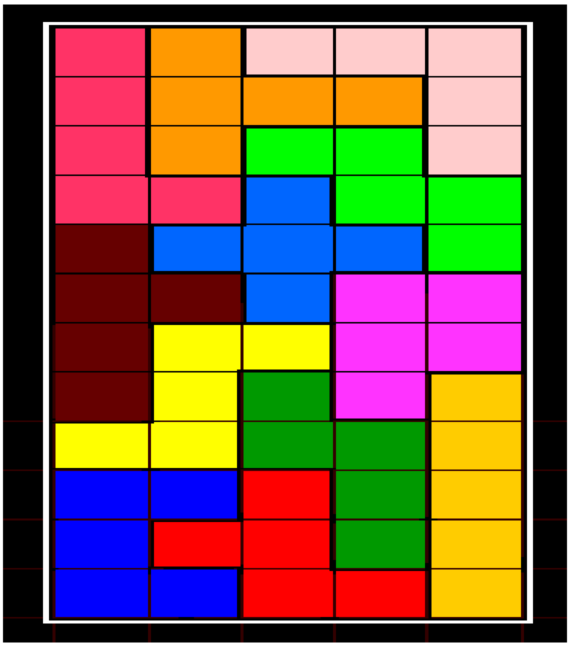
For example: P =  P* = 

Individual Activity:

Using the pentominos grid sheet and safety scissors, cut out the 12 pentominos.

Label each one with its letter. Label any important flipped ones with *.

Using all or some of the pentominos, we can make various shapes. Most of the time, we are interested in making rectangles.



Group Problems: (For Groups of 2/3) or individual competition, fastest to finish the puzzle wins. Using laminated grids and pentomino cutouts from materials sheet at end.

For each problem, encourage students to choose the necessary pieces first and then start to build. There are quite a few problems. If students are finding a particular set too easy, move on in difficulty.

Problem 1: Using 3 Pentominos what size rectangle can we make?

Construct a 3x5 rectangle using

a) L*, N and V,

b) L*, Y and T,

c) L, V and P,

d) U, N* and P*,

e) P, F* and U,

f) U, P* and Y*

g) U, P and V.

Problem 2: Using 4 Pentominos what size rectangle can we make?

Construct a 4x5 rectangle using

a) Y, T, L and F*,

b) L, P, F* and U,

c) L, V, F* and U,

d) Y, P*, U and F*,

e) L, U, V and F

f) I, L*, N and V,

g) L, W, Y and P*,

h) L*, P*, U and Y,

i) L*, W, P* and Y*,

j) L*, W, P and Y*,

k) L*, W, P* and Y,

l) L, T, P* and Y.

We mention that there another 50 similar puzzles of constructing a 4x5 rectangle using four pentominos. Can you find some more?

Problem 3: Using 5 Pentominos what size rectangle can we make?

Construct a 5x5 rectangle

a) Y, T, F, L and I,

b) L, X, F, P and U,

c) L, P, F, U and I,

d) L, V, F, U and I,

e) Y, P*, F, U and I,

f) I, V, L*, U and F.

We mention that there another 170 similar puzzles of constructing a 5x5 rectangle.

Problem 4: Using 6 Pentominos what size rectangle can we make?

Construct a 3x10 rectangle using

a) I, P, N, F, Y* and U,

b) U, F, I, P, Y and V,

c) U, F, N*, Y, P* and V,

d) U, X, Y*, N, P and V.

There are a total of 145 similar puzzles.

Problem 5 (Difficult): Construct a 3x20 rectangle using all the pieces. There are only two solutions here.

Solutions:

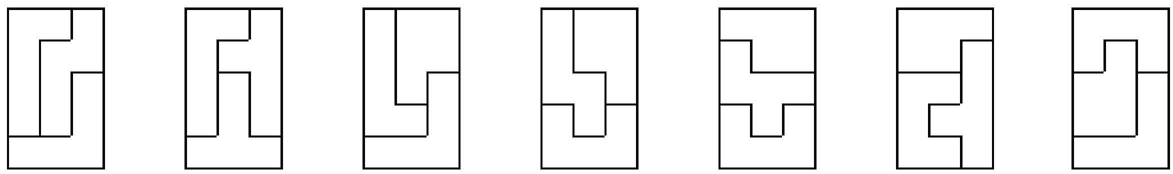
Question: Imagine that the pentominos are fields that we want to fence in. How much fencing do we need to fence each one? Which field is biggest?

Answer: They all need a fence of length 12 except for the P pentomino which needs a fence of length 10. All the fields have the same size, just different shapes.

Problem 1: Using 3 Pentominos what size rectangle can we make?

Answer: 3 pentominos will have area 3x5=15.

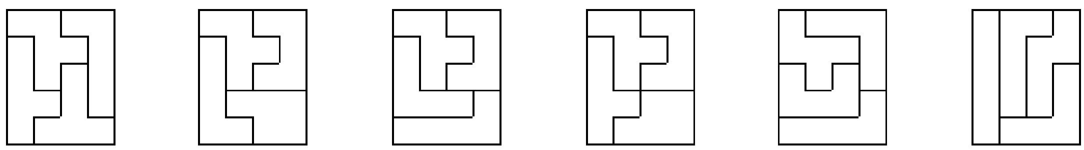
Solutions for the 3x5 rectangles a) to g)



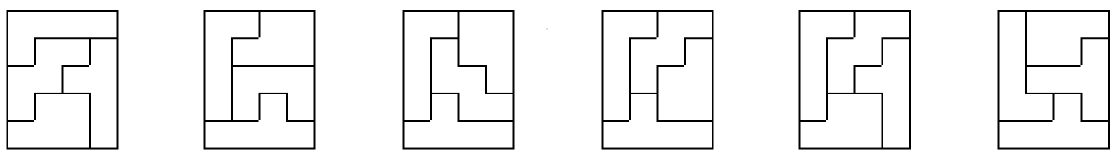
Problem 2: Using 4 pentominos what size rectangles can we make?

Answer: 4 pentominos will have area 4x5=20. 20 can also be written as 2x10 but it's easy to see that this won't work.

Solutions for the 4x5 rectangles a) to f)



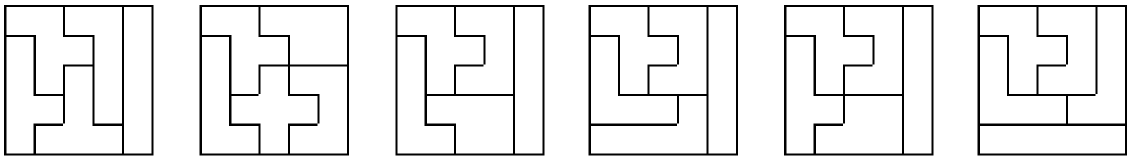
Solutions for the 4x5 rectangles g) to l)



Problem 3: Using 5 pentominos what size rectangle can we make?

Answer: 5 pentominos will have area 5x5=25. 25 has no other factors!

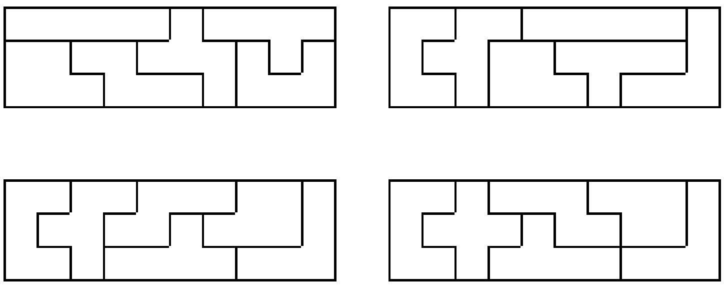
Solutions for the 5x5 rectangles a) to f)



Problem 4: Using 6 pentominos what size rectangle can we make?

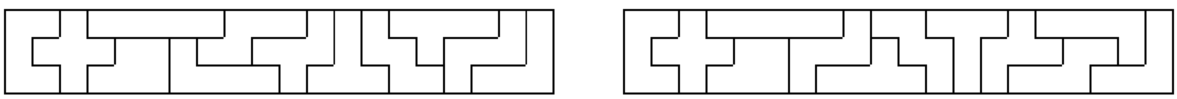
Answer: 6 pentominos will have area 6x5=30. 30 can also be written as 3x10 See if is this possible as a challenge.

Solutions for the 3x10 rectangles a) to d)



Problem 5 (Difficult): Construct a 3x20 rectangle using all the pieces.

Solutions for the 3x20 rectangles



Materials and Templates:

Grid sheets: Print and laminate one of each per group of 2/3 students.

5x5

4x5

6x6

3x5

3x20

(Cut in half along dotted line and tape)

Pentominos grid sheet: Print one per student.