Set Game Wrap Up, Starting Parity Party

Introductory notes:

1. In class review/discussion of some of the Set Game Questions:

Some students like to present their solutions at the board. Encourage them along the way.

- Q2: How many pairs of cards can be made with the cards in the deck? Some groups got

$$80 + 79 + 78 + \cdots + 3 + 2 + 1$$

It looks like a good moment to discuss with everyone why this equals $\frac{80\times 81}{2}$.

- Q6 and on: Ask if anyone thought about these questions. Do as many as we can in the given time.

2. Play 1 round of Set

3. Start Parity Party

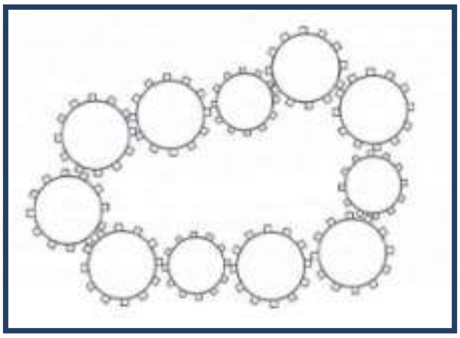
Important: if you use props like dominoes etc. only do it for a very short period of time. They're only needed to demonstrate some thought processes not for full-blown play.

Resources

- Set decks
- Activity sheets per student: Set Game Questions and Parity Party Questions
- A few sets of dominoes.
- M&M boxes – this didn't happen as one of us ate all the M&M's. But the problem solving actually works better with dominoes in place of candy.
- A few chessboards with pieces. (A full set of pieces per board isn't necessary).
- Solution sheets for tutors

Parity Party

1. Gearing up for some maths



In the picture on the right, you see eleven gears in a chain.
Is it possible for all the gears to rotate simultaneously?

What if there was an even number of gears?

2. Odds and Evens

In each of the following sums, find out whether the result is odd or even without actually computing it:

a. $1{,}256{,}827 + 7{,}571{,}269$
b. $999{,}999 - 888{,}888$
c. $10^{10} + 1$
d. 777×256
e. 131×99
f. $5 + 13 + 7 + 21 + 35$
g. $111 + 257 + 549 + 973$
h. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

Let's gather some general rules:

odd+odd=	odd−odd=
even+odd=	even−odd=
even+even =	even−even=
If I add an odd number of odd numbers together, the result is	odd × odd=
If I add an even number of odd numbers, the result is	even × any number=

3. An addition problem

a) If we add up the numbers 1 to 9 we get

1+2+3+4+5+6+7+8+9=45.

However we can change the plus signs to minus signs to get new numbers like

1+2+3+4+5-6-7-8-9=-15 or

-1+2-3+4-5+6-7+8-9=-5

Is it possible to switch some of the plus signs to minus signs so that the numbers add to zero?

b) What if we have the same problem, but we take the numbers 1 to 100 instead?

4. A weighty problem

There is a box full of 1kg, 3kg, and 5kg weights.

a. Is it possible to take exactly 10 weights that together weigh 25kg?

b. I can make 25kg by using:

5 weights:		$5 \times 5 = 25$
7 weights:		$4 \times 5 + 1 \times 3$ $+ 2 \times 1 = 25$

What other numbers of weights can you use to make 25 kg? List them all and explain.

5. A candy problem

You have lots of candies. You make two small piles, and keep the rest as reserve. You can modify the piles as many times as you like by applying these rules in any order:

(i) You can take the same number of candies from each pile.

(ii) You can double the number of candies in a pile.

a) Start with piles of 10 and 3 pieces. Try to finish the game with no candies in each pile. (Don't eat all the candies!)

b) Can you find a way to finish the game with no candies, no matter which numbers of candies you started with in each pile? Try a few examples: 6 and 5 pieces; 9 and 4 pieces.

c) If you start with piles of 10 and 3 pieces, but modify the rules as follows:

(i) You can take the same number of candies from each pile.

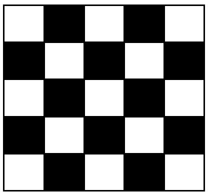
(iii) You can triple the number of candies in a pile.

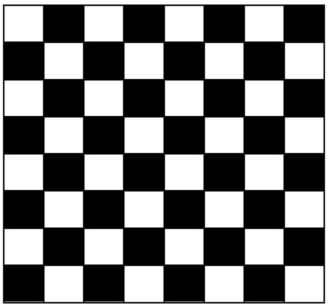
Can you finish the game with no candies in each pile? Explain.

6. Domino problems

a) Is it possible to cover a 5×5 chessboard in dominoes?

What about an 8×8 chessboard?
Explain your answer in both cases.





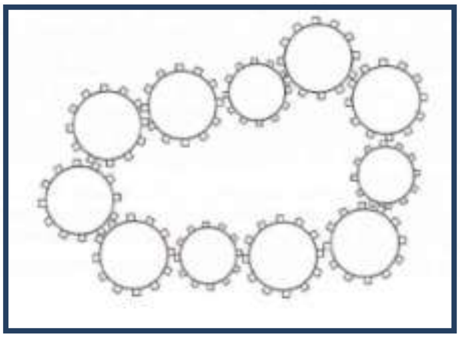
Dominoes:

b) If we take 1 corner square out of a 5×5 chessboard, is it possible to cover the chessboard with dominoes? Explain your answer.

c) If we take two opposite corner squares out of an 8×8 chessboard to make a "mutilated" chessboard, is it possible to cover it in dominoes? Explain your answer.

Parity Party Solutions

1. Gearing up for some maths



In the picture on the right, you see eleven gears in a chain.
Is it possible for all the gears to rotate simultaneously?

What if there was an even number of gears?

Solution:
Pick any gear to start off with, and let's say it rotates clockwise. Then the gear next to it will rotate anti-clockwise, the gear next to that will rotate clockwise and so on. If we move around the chain, we see that the last gear will be rotating clockwise. But this is impossible, because the gear next to it (the first gear) is also rotating clockwise. If there were an even number of gears, they could rotate just fine.

2. Odds and Evens

In each of the following sums, find out whether the result is odd or even without actually computing it:

Even $a. 1{,}256{,}827 + 7{,}571{,}269$
Odd $b. 999{,}999 - 888{,}888$
Odd $c. 10^{10} + 1$
Even $d. 777 \times 256$
Odd $e. 131 \times 99$
Odd $f. 5 + 13 + 7 + 21 + 35$
Even $g. 111 + 257 + 549 + 973$
Odd $h. 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

Let's gather some general rules:

odd+odd=even	odd−odd=even
even+odd=odd	even−odd=odd
even+even =even	even−even=even
If I add an odd number of odd numbers together, the result is odd	odd × odd=odd
If I add an even number of odd numbers, the result is even	even × any number=even

3. An addition problem

a) If we add up the numbers 1 to 9 we get

1+2+3+4+5+6+7+8+9=45.

However we can change the plus signs to minus signs to get new expressions like

1+2+3+4+5-6-7-8-9=-15 or

-1+2-3+4-5+6-7+8-9=-5

Is it possible to switch some of the plus signs to minus signs so that the numbers add to zero?
b) What if we have the same problem, but we take the numbers 1 to 100 instead?

Solution:
a) Since the numbers of odds and evens will always be the same no matter which signs we use, the result will always be odd so not 0.
b) Hint: If this is too difficult, you may start with fewer numbers first. Some may suggest starting with 1, 2, 3, … , 10, but this case isn't any different from a), as it includes 5 odd numbers. You may suggest starting with 8 numbers, so

$$(1-2-3+4)+(5-6-7+8)=0.$$

One possible solution for the numbers from 1 to 100 is:

$$(1-2-3+4)+(5-6-7+8)+\cdots+(97-98-99+100)=0.$$

They might find others.

4. A weighty problem

There is a box full of 1kg, 3kg, and 5kg weights.

a. Is it possible to take exactly 10 weights that together weigh 25kg?
b. I can make 25kg by using:

5 weights: 

$5\times 5=25$

7 weights: 

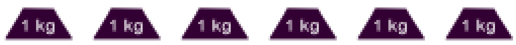
$4\times 5+1\times 3+2\times 1=25$

What other numbers of weights can you use to make 25 kg? List them all and explain.

Solution:
a) The sum of 10 odd numbers will always be even so not 25.
b) We cannot use even numbers of weights as in a).
Hint for the odd numbers: In how many ways can you get 5 kg-s using these weights? Now can you mix and match 5 of these packages?

or

or



We can use any odd number up to (and including) 25.

9 weights: $4\times 5+5\times 1=25$
11 weights: $3\times 5+1\times 3+7\times 1=25$
13 weights: $3\times 5+10\times 1=25$
15 weights: $2\times 5+1\times 3+12\times 1=25$
17 weights: $2\times 5+15\times 1=25$ etc.

You can see the pattern: to increase the number of weights by 2, you can replace the 5kg by 3kg+1kg+1kg or, a 3kg by 1kg+1kg+1kg.

5. A candy problem

You have lots of candies. You make two small piles, and keep the rest as reserve. You can modify the piles as many times as you like by applying these rules in any order:

(i) You can take the same number of candies from each pile.

(ii) You can double the number of candies in a pile.

a) Start with piles of 10 and 3 pieces. Try to finish the game with no candies in each pile. (Don't eat all the candies!)

b) Can you find a way to finish the game with no candies, no matter which numbers of candies you started with in each pile? Try a few examples: 6 and 5 pieces; 9 and 4 pieces.

c) If you start with piles of 10 and 3 pieces, but modify the rules as follows:

(i) You can take the same number of candies from each pile.

(iii) You can triple the number of candies in a pile.

Can you finish the game with no candies in each pile? Explain.

Solution:

a) There are many possibilities. One is:

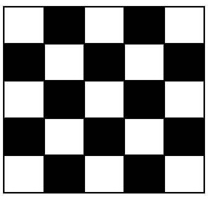
- Double the pile of 3 to get 10 and 6.
- Take away 5 candy from each pile: remain 5 and 1
- Double the 1 to 2 to get 5 and 2
- Subtract 1 from each pile: remain 4 and 1
- Double the 1 to 2 to get 4 and 2
- Double 2 to 4: 4 and 4
- Subtract 4.

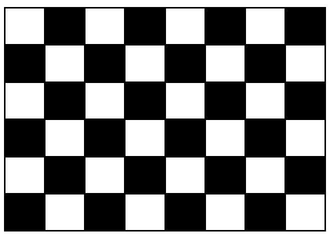
b) One very inefficient strategy is: subtract as many candy as necessary to be left with 1 candy in one of the piles. Then double that 1, subtract 1 from both piles and repeat until all candies are gone.

A more efficient strategy is to double the pile with fewer candies until it's at least half the number of candies in the larger pile. Then you can subtract a certain same number of candies from both piles so that you're left with the smaller pile being exactly half of the bigger pile. We can return to this question in a future session when we discuss solving problems by equations. (More exactly, if there are x candies in the first pile and y candies in the bigger pile, you need to (2x-y) candies from both piles.)

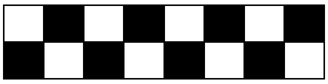
c) Step (iii) doesn't change the parity of the two numbers. Step (i) either preserves parities or swaps them, in either case we'll keep having an even and an odd number. They can never be equal.

6. Domino problems



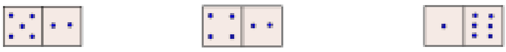


b) Is it possible to cover a 5×5 chessboard in dominoes?



What about an 8×8 chessboard? Explain your answer in both cases.

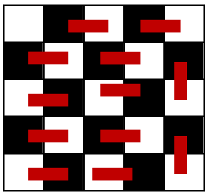
Dominoes:



b) If we take 1 corner square out of a 5×5 chessboard, is it possible to cover the chessboard with dominoes? Explain your answer.
c) If we take two opposite corner squares out of an 8×8 chessboard to make a "mutilated" chessboard, is it possible to cover it in dominoes? Explain your answer.

Solution:
a) You can't split 25 squares into dominoes of 2 squares each.
You can cover the 8×8 chessboard by horizontal dominoes.

b) Yes:



c) Hint:
Answer: No: even though an even number of squares (62) remain, 30 of these are white and 32 are black, while a domino must always cover 1 white and 1 black square, so we can only cover the same number of white and black square.