UCC Maths Enrichment - Polynomials

Tutor: Kieran Cooney **Due:** 14-3-15

Exercises

These problems are designed for you to practice the techniques we developed in class

1. Find all positive integers n for which $n^5 + n^4 + 1$ is a prime.
2. Find a change of variables from x to t that takes the general cubic $x^3 + a\,x^2 + b\,x + c$ to the depressed cubic $t^3 + p\,t + q$.
3. Find all integer solutions to $x^3 - \frac{3}{4}\,x^2 + \frac{8}{3}x + \frac{17}{12} = 0$.
4. Prove the Sophie Germain identity by completing the square in the expression $a^4 + 4b^4$.
5. Find all positive integer solutions to $x\,y + 5\,x + 3\,y = 20$
6. Is the number 102400243 prime?
7. Is $4^{545} + 545^4$ prime?
8. Find a divisor of 999 991 in your head.
9. Find the sum of the cubes of the roots of the polynomial $x^3 + 3x^2 - 7x + 1$
10. Find the sum of the fifth powers of the roots of $x^2 + x + 1$

Problems

Please attempt questions 3, 4 and 9 for the due date.

1. x and y are integers such that $y^2 + 3\,x^2y^2 = 30x^2 + 517$. Find $3x^2y^2$.
2. Prove the polynomial remainder theorem
   If the polynomial $P\,(x)$ is divided by $x - a$ then the remainder will be $P\,(a)$
3. What is the largest integer n for which $n^3 + 100$ is divisible by $n + 10$?
4. The number 27 000 001 has 4 prime divisors. Find their sum.
5. Find all integer solutions (n, m) to
$$n^4 + 2n^3 + 2n^2 + 2n + 1 = m^2$$
6. Show that
$$\frac{(x+y+z)^7 - (x^7 + y^7 + z^7)}{(x+y+z)^3 - (x^3 + y^3 + z^3)}$$
   is a multinomial.
7. Given the polynomial
$$\left(x^{2012} + x^{2011} + 2\right)^{2010}$$
   we may express it as
$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$
   Compute
$$a_0 - \frac{a_1}{2} - \frac{a_2}{2} + a_3 - \frac{a_4}{2} - \frac{a_5}{2} + a_6 - \cdots$$
8. Factor $a^3 + b^3 + c^3 - 3abc$
9. Prove that if $n > 1$ then $n^4 + 4^n$ is composite.
10. Solve the equation $z^8 + 4z^6 - 10z^4 + 4z^2 + 1 = 0$