UCC Mathematics Enrichment - Number Theory

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1. Find the remainder of $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60}$ upon division by 7.
2. Is there any possible way to add up 59 numbers each of which is raised to the power of 60 to get 616161616161616161616161616160?
3. Show that $(x + y)^p \equiv x^p + y^p \, (mod \, p)$, where p is some prime number.
4. For p a prime, conjecture (guess) how many values does x^n take on modulo p, with p a prime? Try proving your conjecture, but it might be difficult.
5. Consider the sequence $a_1, a_2 \ldots$ defined by

$$a_n = 2^n + 3^n + 6^n - 1 \; (n = 1, 2, \ldots)$$

Determine all positive ineters that are relatively prime to every term in the sequence.