Pigeonhole Principle

1. In the following fraction every letter represents a different digit.

$$\frac{B \times L \times U \times E \times B \times E \times R \times R \times Y}{I \times C \times E \times C \times R \times E \times A \times M}$$

Knowing that the value of the fraction is a real number, find its value. Justify your answer!

2. A teacher gives a multiple choice quiz that has 3 questions, each with four possible answers: *a, b, c, d*. What is the minimum number of students that must be in the class in order to guarantee that at least 2 answer sheets will be identical?

Q1	a	b	c	d
Q2	a	b	c	d
Q3	a	b	c	d

3. A row of houses are randomly assigned distinct numbers between 1 and 50 (inclusive). How many houses must there be to insure that there are 5 houses numbered consecutively?

4. Given any 6 integers from 1 to 10, show that some two of them have an odd sum.

5. Prove that having 100 whole numbers, one can choose 15 of them so that the difference of any two is divisible by 7.

6. Show that any subset of 55 distinct positive numbers less than 101 must contain two numbers which differ by exactly 9.

7. One of the numbers -1, 0 or 1 is written in each of the squares of the table:
Prove that among the sums by rows, columns and diagonals there are two equal.

8. Show that if you randomly draw 13 points in the interior of a square of 4 cm sides, then there must exist 4 points among them which are closer than 3 cm from each other.

9. a) A 3×7 rectangular board is divided into 21 squares each of which is coloured red or green. Prove that the board contains a rectangle whose four corner squares are all the same colour.
b) A 4×19 rectangular board is divided into 76 squares each of which is coloured red or green or blue. Prove that the board contains a rectangle whose four corner squares are all the same colour.

10. There are 17 points on the plane, no 3 of which are collinear. The segments that connect the points are coloured in blue, red or green. Prove that among all the triangles thus obtained, there is one all of whose vertices have the same colour.

11. Show that every integer n has a multiple consisting only of 0's and 1's.

12. Show that at a party with at least two people, there are two people who know the same number of other people there. (Assume that if a knows b, then b knows a).

13. The numbers $1, 2, \ldots, 64$ are randomly written in a 8×8 array. Prove that there exists a 3×3 subarray whose numbers have the sum greater than 145.