

Mathematics Enrichment Classes

Exercise Sheet: GCD, the Euclidean Algorithm, and LCM

1. Simplify the following fraction until it is in reduced form:

$$\frac{2015 \times 2014 - 2014 \times 2013 + 2}{310 \times 130}.$$

What is $\gcd(2015 \times 2014 - 2014 \times 2013 + 2, 310 \times 130)$?

2. Prove that the following fractions are not in reduced form:

$$\frac{51}{21}, \quad \frac{501}{201}, \quad \frac{5001}{2001}, \quad \frac{50001}{20001}, \dots, \quad \frac{1 + 5^{k+1} \times 2^k}{1 + 5^k \times 2^{k+1}}$$

3. Find $\gcd(10n + 9, 8n + 7)$ where n is some unknown natural number.

4. Find $\gcd(7^{15} + 1, 7^{12} - 1)$.

5. Find the smallest number that can be written using only the symbols

$$140, \quad 525, \quad +, \quad -$$

any number of times. Prove that your result can indeed be written with those symbols only and that it is the smallest such number.

6. For each of the following pairs of numbers, find their greatest common divisor and write it as an integer linear combination of the two numbers:

i) 39 and 59.

ii) 359 and 2103

7. Find the smallest positive integer in the given set:

$$A = \{48u + 72v : u, v \in \mathbb{Z}\}, \quad B = \{30^{10}u + 10^{30}v : u, v \in \mathbb{Z}\}$$

8. Find all integer solutions to the equations:

i) $17x + 95y = 1$.

ii) $39x + 59y = 1$.

iii) $23x + 101y = 5$.

9. An American wishes to pay \$2.65 using only dimes (10¢) and quarters (25¢). In how many ways is this possible? List them all.

10. A hallway 5 metres long is to be tiled with rows of two sizes of tiles, one of width 8cm and the other of width 18cm. How many different ways are there of choosing the two sizes of tile to carry out this job? List them all.

11. Let a and b be relatively prime. Show that there exists a positive integer x such that ax divided by b yields the remainder 1.

12. Prove that for every n , the number $\gcd(n + 1; n^2 - n + 1)$ is either 1 or 3.

13. Find two mystery numbers knowing that if we divide one by 4 we get the same answer as when we divide the other one by 3, their gcd is 15 and their lcm is 180.